Quality Competition in Network Industries

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ABSTRACT. Access to monopoly segments and interconnection requirements has been analyzed extensively in term of prices, quantities, sunk costs and productivity. This paper considers quality competition in network industries. The aim of this research is to determine the relationship between access to bottleneck network facilities and quality of goods provided by a monopoly and one or several potential competitors when industry network is either regulated or not. Specifically, we determine the relationship between product quality and access charges. We analyze quality competition in a regulated industry under complete and incomplete information regimes.

Concurrence en Qualité dans les Industries de Réseaux

RESUME. - Dans les industries de réseaux, les besoins d'interconnexions et l'accès aux segments monopolisés ont surtout été traité en terme de prix, de quantités, de coûts fixes et de productivité. Ce travail de recherche s'intéresse à la concurrence en qualité dans les industries de réseaux. L'objectif de cet article est de déterminer analytiquement la relation qui existe entre la charge d'accès au segment monopolisé et la qualité des biens offerts par un monopole et par un où plusieurs concurrents potentiels et ce, lorsque l'industrie est régulée ou non. Cette analyse détermine la relation qui existe entre la charge d'accès et les différentes qualités offertes et ce, lorsque l'industrie de réseaux est régulée en régimes d'information complète et incomplète.

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1 Introduction

This paper analyzes quality competition in network industries. Precisely, we attempt to determine the relationship between access to bottleneck network facilities and quality of goods provided by a monopoly and one or several potential competitors when network industry is either regulated or not. Our analysis focuses on telecommunication network industry. Yet, our model applies to several goods and services other than telecommunications services.

Competition within and between networks constitutes a major concern in the network industries literature. Also, it is admitted that the determination of access charges is the fundamental key to the introduction of competition in these industries (Laffont and Tirole, 1994b).

Nonetheless, in reading about this theory, one is struck by the scarcity of analysis about quality provision and competition in these markets. Indeed, access to monopoly segments and interconnection requirements have been analyzed extensively in terms of prices, quantities, sunk costs and productivity.

Hence, the study of quality competition between and within industry networks engender a set of fundamental aspects to be investigated such as:

i) Does there exist an incentive to the monopoly to provide better quality in regulated network industries in particular when competitive pressures may exist?

ii) Should the regulator allow quality competition in and among industry networks?

iii) How to define access rules to the network that creates fair competition in markets where quality variations may exist?

iv) Would regulatory actions be beneficial to the society and what are their main characteristics?

We argue that, on the one hand, quality constitutes a real option for competition within a network and between networks. On the other hand, introducing quality competition in such industries may generate sizable welfare improvements.

It is established that Spence (1975) and Shesinski (1976) have shown that under a monopoly situation the quality provided is suboptimal.
In an unregulated industry network, a monopolist may have two incentives to provide quality: the sale incentives (search goods) and the reputation incentives (experience goods).

These problems have prompted several authors to investigate further the organization of these industries. Indeed, there is an important and recent literature on competition within and between network industries. We refer, among others, to Laffont, Rey and Tirole (1997), Laffont and Tirole (1994a, 1994b), Chakravorti and Spiegel (1994), Cremer, Ivaldi, and Turpin (1996), Klein (1996), Encaoua and Moreaux (1987).

Laffont and Tirole (1994a) analyze access prices in the framework of an optimal regulation under asymmetry of information. The authors develop a theory of how to price access to the bottleneck facility taking into account sunk costs of the network and incentive constraints of the monopoly. The authors show that incomplete information may call for a further departure from marginal cost pricing of access.

Laffont and Tirole (1994b) debate the issue of determination of interconnection charges and their effects on competition in network industries. They put forth a new pricing rule called global price cap.

More recently, Laffont, Rey and Tirole (1997) focus on price competition between two interconnected networks paying access charges to each other. Networks are differentiated as they offer different functions that appeal to different consumers. First, they analyze competition under nondiscriminatory price regime. Except for large access charges or large network substitutability, the authors show the existence of industry equilibrium. Second, the authors show that, in both the mature and the entry phases of the industry, the nature of the competition is substantially affected by the possibility of price discrimination.

Chakravorti and Spiegel (1994) develop a simple model to analyze the conditions under which entry would be attractive from the regulator point of view. They consider the possibility of entry and its effects on quality provided by a regulated firm.

Note that our theoretical framework is closer to the work of Chakravorti and Spiegel (1994). Nonetheless, we depart from this literature on several grounds. First, we develop a model of quality competition in network industries composed by a
regulated monopoly and a new potential entrant. Second, we focus on regulated network industries where potential entrants have to pay a charge to access to the network bottleneck facilities. Third, we analyze quality competition and its impact on the determination of the access charge. Fourth, we focus on the effects of entry on optimal regulation under asymmetric information on quality. Nevertheless, Chakravorti and Spiegel (1994) consider conditions under which entry is attractive from the regulator viewpoint. They study only the complete information benchmark.

In this article, we determine the relationship between product quality and access charges in network industries under alternative informational regimes. A dominant operator controls a bottleneck facility required to interconnect with entrants competing on a complementary segment of the network. Precisely, the dominant operator produces monopolized goods as well as another competitive one that has an imperfect substitute belonging to a competitor. The competitor needs to access to the local network to reach the final consumers. In order to do so, he ought to pay an access charge. In addition, each operator firm produces a different quality.

We show that when the network industry is not regulated, the access charge has a negative effect on the quality provided by the monopolist. If the monopolist imposes a high access charge and a low price, he would not loose his market share. The access charge has also negative consequences on the quality provided by the competitor. So, the higher access charge is, the lower is the competitor quality level.

When the industry is regulated and under complete information, we show three sets of results.

First, the optimal access charge is higher than the marginal cost of the network, because deficits are socially costly. Indeed, if the regulator imposes an access charge inferior to the marginal cost, the prices will be too high and the consumers will buy no goods.

Second, the access charge decreases with the elasticity of the competitor good. In other words, facing an inelastic demand, the competitor could afford to pay the access charge, raise his price without affecting the demand of the consumers.

Third, the higher is the optimal access charge, the higher quality levels of both the monopolist and the competitive goods are. This result implies that the regulator provides incentives the monopolist and the competitor to provide high quality levels.
So, the access charge has a positive impact on quality levels provided by the monopolist and competitor.

Incomplete information affects quality and access charge variables and increases the cost of supplying quality. Since rents are increasing with respect to the level of quality obtained, it is desirable to decrease the access charge and therefore the quality levels of the monopolist and competitor. This is achieved by giving lower incentives for quality than under complete information.

This research will be organized as follows: Section 2 presents the model. Section 3 analyzes complete information equilibrium solutions under both market and regulated industry regimes. Section 4 considers the case of regulated industry under asymmetry of information. Section 5 provides conclusions and further extensions of this work.

2 The Model

A monopolist operates a network and produces a monopolized commodity (good 0) namely a local telephone service, as well as another commodity (good 1), for instance a long distance service. The latter competes with an imperfect substitute commodity (good 2) produced by a rival firm. The dominant operator controls the bottleneck facility required to interconnection with entrants competing on the complementary segment. Therefore, the provision of good 2 by the competing long distance operator requires access to the local network in order to reach the final consumer. So, the rival firm needs to pay an access charge noted a to the monopolist in order to use the bottleneck facility.

Let \( q_i, v_i \) and \( p_i \) indicate respectively the demand, quality and price of good \( i = 0, 1, 2 \). Then, the demand function of good \( i \) \((i = 1, 2)\) could be written as:

\[
q_i = q_i(v_i, v_j, p_i, p_j, \theta) \quad \text{where} \quad i, j = 1, 2 \quad \text{and} \quad i \neq j
\]

Accordingly, consumer demand of goods \( i \) depends on quality levels \( v_1 \) and \( v_2 \), prices \( p_1 \) and \( p_2 \) as well as on a parameter \( \theta \). Moreover, we assume that the demand function of goods \( i \) verifies the following properties:

A1. \( \frac{\partial q_i}{\partial v_j} < 0 \) for \( i, j = 1, 2 \) and \( i \neq j \).
A 2. \( \frac{\partial q_i}{\partial p_i} < 0 \) for \( i = 1,2 \).

A 3. \( \frac{\partial q_i}{\partial p_j} \geq 0 \) for \( i, j = 1,2 \) and \( i \neq j \).

A 1 implies that an increase in quality level of good \( j \) leads to the decrease of demand of good \( i \). A 2 indicates that the demand function of goods \( i \) decreases according to the demand of good \( i \). In A 3, we assume that \( i \) and \( j \) are substitute goods.

Consumers are differentiated according to their taste for quality indicated by a random variable \( \theta \). We let \( F(.) \) be a continuous distribution function (c.d.f.) of \( \theta \) defined on \( [\theta, \bar{\theta}] \) and \( f(.) \) denote a strictly positive probability density function of \( \theta \). The scalar \( F(\theta) \) represents the fraction of consumers with a taste parameter smaller than \( \theta \):

\[
F(\theta) = \begin{cases} 0 & \text{if } \theta < \theta^* \\ 1 & \text{if } \theta > \theta^* \\ f(\theta) & \text{if } \theta \in [\theta, \bar{\theta}] 
\end{cases}
\]

Let \( \beta_0 \) and \( \beta_1 \) designate the efficiency parameters of the incumbent firm respectively on the bottleneck facility and competitive segment, while \( \beta_2 \) indicates that of the rival firm on the competitive segment of the network.

We assume that the provision of quality \( v_i \) is costly. We let \( \Psi(v_i) \) \( (i=0, 1,2) \) be the cost of provision of quality level \( v_i \) of good \( i \) such that:

A 4. \( \Psi' > 0, \Psi'' > 0, \Psi''' \geq 0 \)

A 5. \( \psi(v_2) > \psi(v_1) \).

According to A 4, the cost function of quality provision is convex. Assumption A 5 implies that in order to be able to access to the network, the competitor needs to provide a higher quality than the monopolist on the competitive segment does.

The monopolist cost function could be written as:

\[
CT^M = C^0(\beta_0, Q) + \Psi(v_0) + C^1(\beta_1, q_1) + \Psi(v_1)
\]

where \( C^0(\beta_0, Q) \) and \( C^1(\beta_1, q_1) \) indicate the costs of production of respectively good 0 and good 1 and \( Q \) stands for the aggregate level of network activity:

\[
Q = q_0 + q_1 + q_2
\]

The competitor production cost function is given by:

\[
CT^C = C^2(\beta_2, q_2) + \Psi(v_2)
\]
Where \( C^2(\beta_2, q_2) \) represents the cost of production of good 2 and \( \psi(v_2) \) is the disutility of provision of quality of good 2.

The requirement, that the cost function is linear in quantity and that the marginal cost does not depend on quality, could simply be formalized as:

\[
A6. \quad \frac{\partial^2 C^i}{\partial q_i^2} = 0 \quad \text{where } i = 0, 1, 2.
\]

\[
A7. \quad \frac{\partial^2 C^i}{\partial q_i \partial v_i} = 0 \quad \text{Where } i = 0, 1, 2.
\]

Moreover, we assume the following cost properties:

\[
A8. \quad \frac{\partial q_j}{\partial v_i} > \frac{\partial q_j}{\partial v_j} \quad \text{with } i = 1, 2; \quad j = 1, 2 \quad \text{and } i \neq j.
\]

\[
A9. \quad C_{\beta_i}^i = \frac{\partial C^i}{\partial \beta_i} < 0, \quad C_{q_i}^i = \frac{\partial C^i}{\partial q_i} > 0, \quad C_{v_i}^i = \frac{\partial C^i}{\partial v_i} > 0, \forall i = 0, 1, 2.
\]

Assumption A8 is a technical requirement to ease computations of the access charge. Under hypothesis A9, a firm is more efficient when its cost is lower. Also, it implies that cost structure of a firm is an increasing function with respect to production and quality levels.

### 2.1. The case of an Unregulated Network Industry

In this section, we attempt to determine the relationship between quality levels, market prices and the access charge in the absence of regulatory supervision.

The monopolist wishes to maximize his profit function. The latter could be written as (1):

\[
\Pi^M(p_0, p_1, v_0, v_1) = p_0 q_0 + p_1 q_1 + a q_2 - c^0 (\beta_0, Q) - \psi(v_0) - c^1 (\beta_1, q_1) - \psi(v_1)
\]

This leads to the following first-order conditions:

\[
(2) \quad \frac{p_1 - \frac{\partial C^0}{\partial Q}}{p_1} \left( 1 - \frac{1}{q_2} \frac{\partial q_2}{\partial p_1} \left( \frac{\partial C^0}{\partial Q} - a \right) \right)
\]

\[
(3) \quad p_1 \frac{\partial q_1}{\partial v_1} = \frac{\partial C^0}{\partial Q} \frac{\partial q_1}{\partial v_1} + \frac{\partial C^1}{\partial q_1} \frac{\partial q_1}{\partial v_1} + \psi'(v_1) + \left( \frac{\partial C^0}{\partial Q} - a \right) \frac{\partial q_2}{\partial v_1}
\]

Where \( \eta_1 \) is the price elasticity of good 1.
The competitor wishes to maximize his profit function given by:

\[
\Pi^C(p_2, v_2) = p_2 q_2 - a q_2 - C^2(\beta_2, q_2) - \psi(v_2)
\]

This leads to the following first-order conditions:

\[
p_2 - a = \frac{\partial C^2}{\partial q_2} \frac{\partial q_2}{\partial p_2} = \frac{1}{\eta_2}
\]

\[
p_2 \frac{\partial q_2}{\partial v_2} = \frac{\partial C^2}{\partial q_2} \frac{\partial q_2}{\partial v_2} + \psi'(v_2) + a \frac{\partial q_2}{\partial v_2}
\]

Where \( \eta_2 \) is the price elasticity of good 2.

**Proposition 1:** The equilibrium of the monopolistic firm implies that:

(i) If \( \frac{\partial C^0}{\partial Q} = 0 \) then \( p_1 = p_1^M \) and \( v_1 = v_1^M \),

(ii) If \( \frac{\partial C^0}{\partial Q} > 0 \) then \( p_1 > p_1^M \) and \( v_1 < v_1^M \)

(iii) If \( \frac{\partial C^0}{\partial Q} < 0 \) then \( p_1 < p_1^M \) and \( v_1 > v_1^M \), where \( p_1^M \) and \( v_1^M \) are respectively the monopolist price and quality level.

**Proof:** See appendix

Proposition 1 shows that when access charge is set equal to the marginal cost of the network industry, then the monopolist behavior is not affected by the presence of a competitor in the market. As a result, we have both a price and a quality distortion. The price of good 1 is set higher than the marginal cost and the quality level remains suboptimal.

When access charge is set above the marginal cost, the monopolist finds it beneficial to increase price of good 1 and reduce quality level. The increase in the access charge offsets both the loss of profit due to quality reduction as well as the loss of market share due to price distortion.

In contrast, if the monopolist decides to set access charge lower than the marginal cost of the network, he would then need to decrease the price of good 1 and increase his quality level in order to maintain his market share and generate non-negative profits. But this case is unrealistic.
PROPOSITION 2: The equilibrium of the competitive firm implies that:

(i) The equilibrium price is lower than $p_1^M$: $p_2 < p_1^M$

(ii) The equilibrium quality level is higher than $v_1^M$: $v_2 < v_1^M$

Proof: see the appendix.

According to this proposition, a rational rival firm needs to set its price lower than that of the monopolist. If the competitor increases the price $p_2$ then the demand of good 2 will diminish. Therefore, he needs to decrease his price. The higher is the access charge, the lower the price $p_2$ and the competitor profit are. In order to pay the access charge and obtain positive profits, the competitor should provide quality standards that are lower than those of the monopolist should.

When the industry is not regulated, the access charge has a negative effect on the quality provided by the monopolist. If he imposes a high access charge and a low price, he would not loose any market share. On the other hand, the access charge has a negative impact on the quality provided by the competitor. So, the higher is the access charge, the lower competitor quality level is.

2.1.1. An Example

Consider a network industry with the following cost structure:

$$C^i(\beta_i,q_i) = \beta_i q_i$$

$$\Psi(v) = \frac{\gamma v^2}{2} \quad \text{where} \quad i = 0, 1, 2 \quad \text{and} \quad \gamma > 0.$$

Consumer's preferences could be read as: $U = \theta v - p$ if the consumer purchases a good with quality $v$ and pays a price $p$. He obtains zero utility if he buys no goods. The utility is separable in quality and price. The consumer's taste parameter for quality $\theta$ is distributed uniformly over $[\theta, \theta + 1]$.

Consumers with a taste for quality above $\theta^* = \frac{P_2 - P_1}{v_2 - v_1}$ (indicating consumer's indifference between high-quality and low-quality goods) prefer to buy high-quality
product. However, consumers with a taste for quality less than \( \theta^* = \frac{p_2 - p_1}{v_2 - v_1} \) but above \( \frac{p_1}{v_1} \) prefer to purchase low-quality good. Otherwise, consumers will not buy.

Thus, the demand's functions are given by:

\[
q_1(p_1, p_2, v_1, v_2) = F\left(\frac{p_2 - p_1}{v_2 - v_1}\right) - F\left(\frac{p_1}{v_1}\right)
\]
\[
q_2(p_1, p_2, v_1, v_2) = 1 - F\left(\frac{p_2 - p_1}{v_2 - v_1}\right)
\]

Where \( F \) is the c.d.f. of \( \theta \)

After computation, demands for goods 1 and 2 are as follows:

\[
q_1(p_1, p_2, v_1, v_2) = \frac{p_2 - p_1}{v_2 - v_1} - \theta
\]
\[
q_2(p_1, p_2, v_1, v_2) = \frac{\theta - p_2 - p_1}{v_2 - v_1}
\]

The equilibrium quality level provided by the monopolist is a solution of the following equation:

\[
(7) \quad a = p_1 - \beta_1 - \frac{\nu_1(v_2 - v_1)^2}{p_2 - p_1}
\]

We let \( 7-a \) be the equation that yields the relationship between the access charge and the quality level provided by the monopolist:

\[
(7-a) \quad \frac{\partial a}{\partial v_1} = -\frac{\gamma}{p_2 - p_1} (v_2 - v_1)(v_2 - 3v_1)
\]

The equilibrium quality level for the competitor is given by the following equation:

\[
(8) \quad a = p_2 - \beta_2 - \frac{\nu_2(v_2 - v_1)^2}{p_2 - p_1}
\]

The relationship between the access charge and the quality level provided by the competitor is given by equation \( 8-a \):

\[
(8-a) \quad \frac{\partial a}{\partial v_2} = -\frac{\gamma(v_2 - v_1)(3v_2 - v_1)}{p_2 - p_1}
\]
Equations (7-a) and (8-a) show immediately that the access charge has a negative effect on both quality levels of the competitive good and the monopoly one 
\[ \frac{\partial a}{\partial v_2} < 0 \text{ and } \frac{\partial a}{\partial v_1} < 0 \].

According to the previous results, it is obvious that in the absence of regulatory supervision, the monopolist has an incentive to provide a quality level that maximizes the access charge in order to gain a larger market share. Therefore, we have three major implications: First, monopoly quality level is sub-optimal which is compatible with the general theory of monopolistic behavior as in Spence (1975) and Shesinski (1976). Indeed, if there is no competitor, the monopolist will have the market power and will impose a sub-optimal quality level. Second, there will be no diversity of products. When there is no competition, product differentiation is low. Third, quality level will also be low even in the presence of competitive pressures. Hence, we have shown that when the access charge increases, quality levels of both the competitor and monopolist decrease. The competitor will not provide a high quality level because of the monopolist's market power.

The regulation of a network industry is justified on two grounds. On the one hand, a regulator should facilitate access of firms to the bottleneck facility. On the other hand, he must rationalize the access charge. Indeed, the regulator must impose an access charge that is not excessively high in order to allow access to the competitor which in turn makes him compete on the product market and provide a high quality level.

2.2. A Regulated Network Industry under complete information

We let \( t \) indicate the net transfer received by the monopolist from the regulator. We suppose that the regulator reimburses costs to the monopolist, receives directly the revenue from the sale of the competitive good and network good to the consumers. The monopolist receives also the access charge. Therefore, after observing the quality of the monopolized good (which is identified by the taste parameter), the regulator decides to make a transfer to the monopolist in order to subsidize the access of the competitor to the bottleneck facility and thus allows him
to provide a high quality level.

So, the monopolist profit is given by:

\[ \Pi^M(p_1, v_1) = t + ap_2 - \psi(v_0) - \psi(v_1) \]

The competitor profit is:

\[ \Pi^C(p_2, v_2) = p_2 q_2 - C^2(\beta_2, q_2) - \psi(v_2) - a q_2 \]

The consumers' utility is given by the following expression:

\[ U(q_0, q_1, q_2, v_0, v_1, v_2) = S(q_0) + V(q_1, q_2) - p_0 q_0 - p_1 q_1 - p_2 q_2 \\
- (1 + \lambda)(t + C^0(\beta_0, Q, v_0) + C^1(\beta_1, q_1, v_1) - p_0 q_0 - p_1 q_1) \]

Where \( S(q_0) \) is consumers' utility for good 0, \( V(q_1, q_2) \) is consumers' utility for commodity 1 and 2, and \( (1 + \lambda) \) is the shadow cost of public funds (\( \lambda > 0 \) because of distortionary taxation). We assume that the functions \( S \) and \( V \) are concave and that the cost functions are convex.

Then, the welfare function is given by:

\[ W(p_1, p_2, v_1, v_2) = U(q_0, q_1, q_2, v_0, v_1, v_2) + \Pi^M(p_1, v_1) + \Pi^C(p_2, v_2) \]

The regulator wishes to determine the optimal access charge. So he maximizes the following social welfare function given the rationality constraints of both the monopolist and the competitor:

\[ Max_{p_i, v_i} W(p_1, p_2, v_1, v_2) \]

subject to:

\[ \Pi^M \geq 0, \]
\[ \Pi^C \geq 0 \]

Where \( i = 1,2 \).

Therefore, if the industry is regulated, the first order conditions on qualities and prices are the followings:

\[ \frac{p_1 - C^0_0 - C^1_0}{p_1} = \frac{\lambda}{\lambda + 1} \frac{1}{\eta_1} \]

\[ \frac{p_2 - C^0_2 - C^2_2}{p_2} = \frac{\lambda}{\lambda + 1} \frac{1}{\eta_2} \]

\[ 2\psi'(v_1) = p_1 \frac{\partial q_1}{\partial v_1} + p_2 \frac{\partial q_2}{\partial v_1} - \frac{\partial q_2}{\partial v_1} \frac{\partial C^2}{\partial q_2} - \frac{\partial Q}{\partial v_1} \frac{\partial C^0}{\partial Q} - \frac{\partial q_1}{\partial v_1} \frac{\partial C^1}{\partial q_1} \]
\[
\psi'(v_2) = (1 + \lambda) \left[ p_1 \frac{\partial q_1}{\partial v_2} + p_2 \frac{\partial q_2}{\partial v_2} - \frac{\partial q_2}{\partial v_2} \frac{\partial C^2}{\partial q_2} - \frac{\partial Q}{\partial v_2} \frac{\partial C^0}{\partial q_2} - \frac{\partial q_1}{\partial v_2} \frac{\partial C^1}{\partial q_1} \right]
\]

Where \( \eta_1, \eta_2 \) are super-elasticity of good 1 and 2.

Equation (13) (resp. (14)) ensures equality between the Lerner index of the good 1 (respectively 2) and the Ramsey index.

Then, the monopolist sets the equilibrium access charge given by:

\[
a = p_2 - \frac{\partial C^2}{\partial q_2}
\]

So using equation (14), the access charge will be written as:

\[
a = C^0_Q + \frac{\lambda}{\lambda + 1} \frac{p_2}{\eta_2}
\]

**PROPOSITION 3:** When the industry is regulated and firms compete in quality, then the optimal access charge depends both on the efficiency of the firm and quality level provided:

\[
a^C = H(C^0_Q, \eta_2, C^2_Q) + K(\psi'(v_1), \psi'(v_2), q_{1v_1}, q_{2v_2}, q_{1v_1}, q_{2v_2})
\]

where \( H \) is a function of efficiency and \( K \) is a function of quality level and

\[
q_{iv_1} = \frac{\partial q_i}{\partial v_1} \quad q_{iv_j} = \frac{\partial q_i}{\partial v_j} \quad \text{with } i = 1, 2; j = 1, 2; \text{and } i \neq j.
\]

\( a^C \) is the access charge under complete information.

As a result, we have:

(i) The optimal access charge is higher than the marginal cost of the network \( C^0_Q \).

(ii) The higher the optimal access charge is, the lower the elasticity of good 2 is.

(iii) The higher the optimal access charge is, the higher quality levels of goods 1 and 2 are.

**Proof:** see appendix

First, the optimal access charge is greater than the marginal cost of the network \( C^0_Q \) because deficits are socially costly. Indeed, if the regulator imposes an access charge inferior to the marginal cost, the prices will be too high and the consumers will buy no goods.

Second, the access charge decreases with the elasticity of good 2. Indeed, if the competitor faces a demand that is inelastic, he can pay the access charge and raise his price without affecting the demand of the consumers.

Third, the higher is the optimal access charge, the higher quality levels of good 1 and good 2. This result implies that the regulator provides incentives to the
monopolist and the competitor to provide high quality levels. So, under a complete information regulated industry, the access charge has a positive impact on quality levels provided by both the monopolist and competitor. This is not the case in the previous section.

In the previous example, the optimal access charge in a regulated industry is given by:

\[
a = \beta_0 \left[ \frac{\lambda}{\lambda + 1} \frac{p_2(p_1^2 q_2 + q_1)}{(p_1 p_2)^2 - 1} \right] + \frac{\lambda}{\lambda + 1} \frac{p_2(p_1^2 q_2 + q_1)}{(p_1 p_2)^2 - 1} \left[ \beta_2 + \frac{2 \gamma \nu_2^3}{\lambda + 1} v_2^2 \right] \theta(v_1^2 v_2^2 - 1)
\]

We can see immediately that the access charge \(a\) increases with \(v_1\) and \(v_2\).

3 Quality Provision under Asymmetric Information

In this section, we examine the impact of the access charge on the quality levels provided by the monopolist and by the competitor when the regulator has an imperfect information on the taste parameter and quality level provided by the monopolist. In the previous section, we have shown that the access charge increases with quality levels of goods provided by both the monopolist and competitor. One can think that the regulator should impose an access charge that decreases with quality level of good 2. But, if this is the case, the competitor will find it beneficial not to reveal the true quality level of his good. However, by imposing an access charge that increases with the quality of the good provided by the competitor, the regulator links the access of the competitor to the network and the provision of higher quality services than that provided by the monopolist. Hence, the regulator ensures both the efficiency of the competitor and true announcement. The monopolist will also have incentives to announce a quality level that is higher than he really provides in order to receive higher transfers and access charge. Therefore, the duty of the regulator is to control this monopolistic informational rent due to private information.

We suppose that the costs function \(C(.)\) and \(\Psi_i(v_i)\) (where \(i=1,2\)) are known by the regulator and that he knows the distribution of \(\theta\). Quality variables \(v_i\) (\(i=1,2\)) and the taste parameter \(\theta\) are private information of each firm and are not observable by the regulator.
We have a problem of regulatory game under adverse selection and moral hazard. If we suppose that the regulator observes the prices \( p_1 \) and \( p_2 \) and the quantities \( q_1 \) and \( q_2 \), we can reduce this problem to a Principal-Agent model with adverse selection. Indeed, the regulator has the possibility to deduce the quality level of the monopolist through the announcement of the taste parameter by the monopolist and observability of prices and quantities as shown in the following relation inferred from the respective demand functions \( q_1(p_1, p_2, v_1, v_2, \theta) \) and \( q_2(p_1, p_2, v_1, v_2, \theta) \) of the monopolist and competitor:

\[
v_1 = \phi_1(p_1, p_2, q_1, q_2, v_2, \theta)
\]

where \( \phi_1 \) is the quality level of a type \( \theta \) monopolist.

We verify that \( \frac{\partial \phi_1}{\partial \theta} < 0 \) whenever the monopolist announces a taste parameter \( \theta \) lower than his true characteristic, he needs to increase his quality level. If this is not the case, the regulator will then understand that the monopolist is lying.

Appealing to the revelation principle, we consider a revelation mechanism:

\[
\{t(\tilde{\theta}), C_0(\tilde{\theta}), C_1(\tilde{\theta}), q_1(\tilde{\theta}), q_2(\tilde{\theta}), Q(\tilde{\theta}), a(\tilde{\theta})\}
\]

This mechanism can be interpreted as contracts that specify the transfer received, the costs to achieve, the quantities to produce and the access charge if the firm announces a characteristic \( \tilde{\theta} \).

The game between the regulator and the monopolist is as follows: first, the monopolist announces his type \( \tilde{\theta} \) to the regulator. Second, the regulator proposes the contract defined by the revelation mechanism. Thus, the monopolist chooses his quality level that cannot be different from \( \phi_1 \). Third, the regulator observes prices and quantities. Finally, the contract is realized and the monopolist receives his transfer. We suppose that:

\[
\frac{\partial^2 \phi_1}{\partial \theta \partial v_2} = 0.
\]

Let \( \Pi^M(\theta, \tilde{\theta}) \) be the profit of the monopolist of type \( \theta \) who announces that his type is \( \tilde{\theta} \). Then, the mechanism must satisfy the incentive constraint in order to make the monopolist tell the truth:

\[
\Pi^M(\theta, \theta) > \Pi^M(\theta, \tilde{\theta}) \quad \forall \theta, \tilde{\theta} \in [\theta, \tilde{\theta}]
\]

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The mechanism must satisfy the individual rationality constraint in order to allow the monopolist to take part in this mechanism:

\[
\Pi^M(\theta) = \Pi^M(\theta, \theta) \geq 0 \quad \forall \theta \in [0, \bar{\theta}]
\]

Under the regulatory mechanism, the monopolist chooses his announcement \(\tilde{\theta}\) in order to maximize his utility:

\[
\max_{\tilde{\theta}} \Pi^M(\theta, \tilde{\theta}) \Rightarrow \frac{\partial \Pi^M(\theta, \tilde{\theta})}{\partial \tilde{\theta}} = 0 \quad \text{pour} \ \tilde{\theta} = \theta
\]

If condition (21) is satisfied and given that:

\[
\Pi^M(\theta, \tilde{\theta}) = t(\tilde{\theta}) + a(\tilde{\theta})q_2(\tilde{\theta}) - \psi(v_0) - \psi(\phi_1(p_1, p_2, q_1, q_2, v_2, \theta))
\]

where only the prices and quantities depend on the announcement of the monopolist, we can deduce a simpler incentive constraint:

\[
\frac{\partial \Pi^M(\theta)}{\partial \theta} = -\frac{\partial \phi_1}{\partial \theta} \psi'(v_1(\theta))
\]

where

\[
\Pi^M(\theta) = t + aq_2 - \Psi(v_0) - \Psi(\phi_1(p_1, p_2, q_1, q_2, v_2, \theta))
\]

Given that \(\frac{\partial \phi_1}{\partial \theta} < 0\), the rent is increasing in \(\theta\). The rationality constraint is equivalent to:

\[
\Pi^M(\theta) = 0.
\]

The regulator program is expressed as follows:

\[
\text{Max}_{p_1, q_1, v_1} W(p_1, p_2, q_1, q_2, v_2, \theta)
\]

Subject to

\[
\frac{\partial \Pi^M(\theta)}{\partial \theta} = -\frac{\partial \phi_1}{\partial \theta} \psi'(v_1(\theta)) \quad (\text{IC})
\]

\[
\Pi^M(\theta) = 0 \quad (\text{IR})
\]

Thus regulator maximizes his social welfare function\(^1\) given by (25):

---

\(^1\) For technical details, see the Appendix.
\[ W(p_1, p_2, v_1, v_2, \theta) = \frac{1}{2} \left\{ S(q_0) + V(q_1(p_1, p_2, v_1(\theta), v_2(\theta)): q_2(p_1, p_2, v_1(\theta), v_2(\theta))) + \lambda p_0 q_0 + \lambda p_1 q_1(p_1, p_2, v_1(\theta), v_2(\theta)) + \lambda p_2 q_2(p_1, p_2, v_1(\theta), v_2(\theta)) \right. \\
\left. - (1 + \lambda) \left[ C^0(\beta_0, Q(\theta)) + C^1(\beta_1, q_1(\theta)) + C^2(\beta_2, q_2(\theta)) + \Psi(v_0) + \Psi(v_1(\theta)) \right] \right\} \\
- \psi(v_2(\theta)) - \lambda U(\theta) \, dF(\theta) \]

subject to the incentive constraint of the incumbent firm:

\[ \frac{\partial \Pi^M(\theta)}{\partial \theta} = - \frac{\partial \phi_1}{\partial \theta} \psi'(v_1(\theta)) \] (I.C.)

and the rationality constraint of the incumbent firm:

\[ \Pi^M(\theta) = 0. \] (I.R.)

Under asymmetric information, the optimal access charge is given by:

\[ a^I = C_{\theta 0} + \frac{\lambda}{\lambda + 1} \frac{p_2}{\eta_2} + \frac{\lambda}{\lambda + 1} \frac{F}{f} \psi'(v_1) \left[ \frac{\partial q_1}{\partial \theta}, \frac{\partial^2 \phi_1}{\partial \theta \partial \phi_2} - \frac{\partial q_1}{\partial \theta}, \frac{\partial^2 \phi_1}{\partial \theta \partial \phi_1}, \frac{\partial q_2}{\partial \theta}, \frac{\partial q_1}{\partial \theta}, \frac{\partial q_2}{\partial \phi_1}, \frac{\partial \phi_1}{\partial \phi_2} \right] \]

**PROPOSITION 4:** If the network is regulated, and when there is quality competition under asymmetric information on the quality level of the monopolist \((v_1)\) and on the taste parameter \(\theta\), the optimal access charge is given by:

\[ a^I = a^C + I(\psi'(v_1), \frac{\partial \phi_1}{\partial \theta}, q_{1v_1}, q_{2v_2}, q_{i1}, q_{i2}, q_{j1}, q_{j2}) \]

where \(q_{iv_i} = \frac{\partial q_i}{\partial v_i}\) and \(q_{iv_j} = \frac{\partial q_i}{\partial v_j}\) with \(i, j = 1, 2\) and \(i \neq j\).

1 is a function depending on incentive constraints \((I(., < 0)).\)

\(a^I\) is the access charge under incomplete information.

As a result we have:

(i) Uncertainty lowers the access charge compared to that of complete information.
(ii) The higher the access charge is, the lower the monopoly quality level is. Uncertainty lowers the monopoly quality level compared to that of complete information.
(iii) The higher the access charge is, the lower the competitor quality level is. Uncertainty lowers the competitor quality level compared to that of complete information.

**Proof:** see the appendix.

Therefore, uncertainty affects both quality and access charges and increases the cost of quality provision. Since rents are increasing with respect to the quality obtained, it

\[ 2 \text{ For the details of computations, see the appendix.} \]
is desirable to lowers both access charge and quality levels of the monopolist and competitor. This result is achieved by giving lower incentives for quality than under complete information.

4 Conclusion

When the industry is not regulated, the access charge has a negative effect on the quality provided by the monopolist. If the monopolist imposes a high access charge and a low price, he would not loose his market share. On the other hand, the access charge has a negative impact on the quality provided by the competitor. So, the higher access charge is, the lower is the competitor quality level.

When the industry is regulated and under complete information, we show first, the optimal access charge is superior to the marginal cost of the network $C_0^0$ because deficits are socially costly. Indeed, if the regulator imposes an access charge inferior to the marginal cost, the prices will be too high and the consumers will buy no goods.

Second, the access charge decreases with the elasticity of the competitor good. Indeed, if the competitor faces a demand which is inelastic, he could pay the access charge, raise his price without affecting the demand of the consumers.

Third, the higher the optimal access charge is, the higher quality levels of both the monopolist and the competitive goods are. This result implies that the regulator provides incentives the monopolist and the competitor to provide high quality levels. So, in a regulated industry and under complete information, the access charge has a positive impact on quality levels provided by the monopolist and competitor.

Incomplete information affects the quality and access charge variables and increases the cost of supplying quality. Since rents are increasing with respect to the level of quality obtained, it is desirable to decrease the access charge and therefore the quality levels of the monopolist and competitor. This is achieved by giving lower incentives for quality than under complete information.
APPENDIX

• Proof of Propositions 1 and 2
The monopolist objective function is:
\[ \Pi^M (p_1, q_1) = p_1 q_1 + p_1 q_2 - a q_2 - C (p_1 q_1) - C (p_1 q_2) \]

First order profit maximization condition yields:
\[ \frac{\partial \Pi^M (p_1, q_1)}{\partial q_1} = 0 \]
\[ = p_1 + q_1 \frac{\partial q_1}{\partial q_1} + q_1 \frac{\partial q_2}{\partial q_1} + q_1 = 0 \]
\[ \Rightarrow q_1 = \frac{-p_1 + q_1 \frac{\partial q_1}{\partial q_1} + q_1 \frac{\partial q_2}{\partial q_1} + q_1}{p_1} \]

Setting \( q_1 = \frac{-p_1}{p_1} \), then we obtain easily equation (2).

Similarly, we have:
\[ \frac{\partial \Pi^M (p_1, q_1)}{\partial q_2} = 0 \]
\[ = p_1 + q_1 \frac{\partial q_1}{\partial q_2} + q_2 \frac{\partial q_2}{\partial q_2} + q_2 = 0 \]
\[ \Rightarrow q_2 = \frac{-p_1 - q_1 \frac{\partial q_1}{\partial q_2} + q_2}{p_1} \]

This gives the result (3).

Consider the profit function of the given by:
\[ \Pi^C (p_2, q_2) = p_2 q_2 - a q_2 - C (p_2 q_2) \]

Profit maximization first order conditions yields equations (5) and (6).

• Proof of Proposition 3

1 Price Determination:
First order welfare maximization conditions with respect to (p1, p2) yield:
\[ \frac{\partial \nu}{\partial q_1} + \frac{\partial \nu}{\partial q_2} + \lambda \left[ q_1 \frac{\partial q_1}{\partial q_1} + p_2 \frac{\partial q_2}{\partial q_1} \right] = 0 \]

\[ = (1 + \lambda) \left[ \frac{\partial \nu}{\partial q_1} + \frac{\partial \nu}{\partial q_2} + \lambda \left[ q_1 \frac{\partial q_1}{\partial q_1} + p_2 \frac{\partial q_2}{\partial q_1} \right] = 0 \]

\[ \frac{\partial \nu}{\partial q_2} + \frac{\partial \nu}{\partial q_2} + \lambda \left[ q_2 + p_2 \frac{\partial q_2}{\partial q_2} \right] = 0 \]

\[ = (1 + \lambda) \left[ \frac{\partial \nu}{\partial q_1} + \frac{\partial \nu}{\partial q_2} + \lambda \left[ q_2 + p_2 \frac{\partial q_2}{\partial q_2} \right] = 0 \]

If \( p_1 = \frac{\partial \nu}{\partial q_1} \) and \( p_2 = \frac{\partial \nu}{\partial q_2} \), we have the following linear equation system:

\[ \begin{bmatrix} q_1 & q_2 & 1 \\ q_1 & q_2 & 1 \\ q_2 & q_2 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial q_1}{\partial q_1} \\ \frac{\partial q_1}{\partial q_2} \\ \frac{\partial q_2}{\partial q_1} \end{bmatrix} = \begin{bmatrix} \lambda q_1 \\ \lambda q_2 \end{bmatrix} \]

Using Cramer rule, we get the following equations:
Equation (a) and (b) give equilibrium prices \((p_1, p_2)\) as functions of quality. Thus, we have (13) and (14).

2 Determination of Equilibrium Quality:

Maximizing social welfare function with respect to \((v_1, v_2)\) yields:

\[
2\psi'(v_1) - p_1 \frac{\partial q_1}{\partial v_1} - p_2 \frac{\partial q_2}{\partial v_1} + \frac{\partial q_1}{\partial v_1} + \frac{\partial q_2}{\partial v_1} + \frac{\partial q_1}{\partial v_2} + \frac{\partial q_2}{\partial v_2} = 0
\]

\[
\psi'(v_2) - (1 + \lambda) \left[ p_1 \frac{\partial q_1}{\partial v_2} + p_2 \frac{\partial q_2}{\partial v_2} \right] = 0
\]

These two equations are (15) and (16). The latter can be written in matrix form as follows:

\[
\begin{bmatrix}
\frac{\partial q_1}{\partial v_1} \\
\frac{\partial q_2}{\partial v_1} \\
\frac{\partial q_1}{\partial v_2} \\
\frac{\partial q_2}{\partial v_2}
\end{bmatrix}
= \frac{1}{p_1}
\begin{bmatrix}
\frac{\partial q_1}{\partial v_1} + \frac{\partial q_2}{\partial v_1} + \frac{\partial q_1}{\partial v_2} + \frac{\partial q_2}{\partial v_2} \\
\frac{\partial q_1}{\partial v_1} + \frac{\partial q_2}{\partial v_1} + \frac{\partial q_1}{\partial v_2} + \frac{\partial q_2}{\partial v_2}
\end{bmatrix}
\]

Using Cramer rule, we obtain equations (a) and (b):

\[
p_1 = C_1 \frac{1}{q_1} + C_2 \frac{1}{q_2}
\]

\[
p_2 = C_1 \frac{1}{q_1} + C_2 \frac{1}{q_2}
\]

Equation (a) and (b) give equilibrium prices \((p_1, p_2)\) as functions of quality.

Substituting equation (b) in (18), gives the optimal access charge given by:

\[
a^C = \frac{C_0}{Q} \left[ q_1 + \frac{1}{\lambda + 1} q_1 + \frac{1}{\lambda + 1} q_2 \right]
\]

If we set:

\[
H(C_0, \bar{q}_1, C_2) = C_0 \left[ 1 + \frac{1}{\lambda + 1} q_1 + \frac{1}{\lambda + 1} q_2 \right]
\]

and

\[
K(\psi'(v_1), \psi'(v_2), q_{1v}, q_{2v}, q_{1u}, q_{2u}) = \frac{\lambda}{\lambda + 1} \left[ \frac{2\psi'(v_1) \frac{\partial q_1}{\partial v_1}}{q_1} + \frac{\psi'(v_2) \frac{\partial q_2}{\partial v_1}}{q_2} \right]
\]

we obtain:

\[
a^C = H(C_0, \bar{q}_1, C_2) + K(\psi'(v_1), \psi'(v_2), q_{1v}, q_{2v}, q_{1u}, q_{2u})
\]

•Proof of Proposition 4
1 - Determination of equation (25)

In this appendix, we provide details of computations that yields equation social welfare. Indeed, the social welfare function is the sum of consumer surplus, monopolist and competitor profit, such that we have:

\[
W = \frac{\partial}{\partial \theta} \{ S(q_0) + V(q_1(p_1, p_2, v_1(\theta), v_2(\theta))); g_2(p_1, p_2, v_1(\theta), v_2(\theta)) + \lambda q_1 q_1 + \lambda p_2 q_2 \}
\]

\[-(1 + \lambda) \left[ C^0(\beta_0, Q(\theta)) + C^1(\beta_1, q_1(\theta)) + C^2(\beta_2, q_2(\theta)) + \Psi(v_0) + \Psi(v_1) \right] - \Psi(v_2) - \lambda U(\theta) \}
\]

We know that the information parameter \( \theta \) has a density function \( f(\theta) \) for \( \theta \in [\theta, \bar{\theta}] \).

Therefore, we have the equation of social welfare:

\[
W = \frac{\partial}{\partial \theta} \{ S(q_0) + V(q_1(p_1, p_2, v_1(\theta), v_2(\theta))); g_2(p_1, p_2, v_1(\theta), v_2(\theta)) + \lambda q_1 q_1 + \lambda p_2 q_2 \}
\]

\[-(1 + \lambda) \left[ C^0(\beta_0, Q(\theta)) + C^1(\beta_1, q_1(\theta)) + C^2(\beta_2, q_2(\theta)) + \Psi(v_0) + \Psi(v_1) \right] - C^2(\beta_2, q_2(\theta)) - \Psi(v_2(\theta)) \}
\]

2 - Proof of equation (26)

Determination of optimal prices and access charge under asymmetric information:

Let \( \mu(\theta) \) be the state variable relative to \( U \).

Using the Pontryagin principle, we have:

\[
\frac{\partial \mu(\theta)}{\partial \theta} = \lambda f(\theta).
\]

Using transversality condition, \( \mu(\bar{\theta}) = 0 \), we have:

\[
\mu(\theta) = \lambda F(\theta),
\]

The Hamiltonian of the optimization program is given by:

\[
H = \{ S(q_0) + V(q_1(p_1, p_2, v_1(\theta), v_2(\theta))); g_2(p_1, p_2, v_1(\theta), v_2(\theta)) + \lambda q_1 q_1 + \lambda p_2 q_2 \}
\]

\[-(1 + \lambda) \left[ C^0(\beta_0, Q(\theta)) + C^1(\beta_1, q_1(\theta)) + C^2(\beta_2, q_2(\theta)) + \Psi(v_0) + \Psi(v_1) \right] - \Psi(v_2(\theta)) - \lambda U(\theta) \}
\]

Differentiating the Hamiltonian with respect to prices \( p_1 \) and \( p_2 \), yields the following equations:

\[
(1 + \lambda) \frac{\partial q_1}{\partial p_1} p_1 + (1 + \lambda) \frac{\partial q_2}{\partial p_2} p_2 = (1 + \lambda) f(\theta) \left[ \frac{\partial Q}{\partial p_1} \frac{\partial C^0}{\partial Q} + \frac{\partial q_1}{\partial p_1} \frac{\partial C^1}{\partial q_1} + \frac{\partial q_2}{\partial p_2} \frac{\partial C^2}{\partial q_2} \right] - \lambda q_1 + \lambda F(\theta) \Psi(v_1) \left[ \frac{\partial^2 \phi_1}{\partial \theta \partial p_1} + \frac{\partial \phi_1}{\partial p_1} \frac{\partial^2 \phi_1}{\partial \theta \partial \theta} \right].
\]
(1 + \lambda) \frac{\partial q_1}{\partial p_1} p_1 + (1 + \lambda) \frac{\partial q_2}{\partial p_2} p_2 = (1 + \lambda) f(\theta) \left[ \frac{\partial Q}{\partial p_2} \frac{\partial C_0}{\partial \phi} + \frac{\partial q_1}{\partial p_2} \frac{\partial C_1}{\partial \phi} + \frac{\partial q_2}{\partial p_2} \frac{\partial C_2}{\partial \phi} \right] - \lambda q_2 + \lambda F(\theta)'(v_1) \left[ \frac{\partial^2 q_2}{\partial \phi^2} + \frac{\partial q_1}{\partial p_2} \frac{\partial^2 \phi}{\partial \phi \partial \phi_1} \right]

Using a matrix format, we could write the above equations as follows:

$$\begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \frac{\partial q_2}{\partial p_2} \\ \frac{\partial q_1}{\partial \phi} & \frac{\partial q_2}{\partial \phi} \\ \frac{\partial q_1}{\partial \phi_1} & \frac{\partial q_2}{\partial \phi_2} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} (1 + \lambda) f(\theta) \left[ \frac{\partial Q}{\partial p_2} \frac{\partial C_0}{\partial \phi} + \frac{\partial q_1}{\partial p_2} \frac{\partial C_1}{\partial \phi} + \frac{\partial q_2}{\partial p_2} \frac{\partial C_2}{\partial \phi} \right] - \lambda q_2 + \lambda F(\theta)'(v_1) \left[ \frac{\partial^2 q_2}{\partial \phi^2} + \frac{\partial q_1}{\partial p_2} \frac{\partial^2 \phi}{\partial \phi \partial \phi_1} \right] \end{bmatrix}$$

Resolving this program, we obtain the following equations:

$$\frac{p_1 - C_0}{Q} q_1 = \frac{\lambda}{\lambda + 1} \frac{1}{\eta_1} + \frac{\lambda}{\lambda + 1} F \left( v_1 \right) \frac{\Psi'(v_1)}{p_1} \left[ \frac{\partial q_1}{\partial \phi_1} \frac{\partial^2 \phi}{\partial \phi \partial \phi_1} + \frac{\partial q_2}{\partial \phi_1} \frac{\partial^2 \phi}{\partial \phi \partial \phi_2} + \frac{\partial q_2}{\partial \phi_2} \frac{\partial^2 \phi}{\partial \phi \partial \phi_1} \right]$$

$$\frac{p_2 - C_0}{Q} q_2 = \frac{\lambda}{\lambda + 1} \frac{1}{\eta_2} + \frac{\lambda}{\lambda + 1} F \left( v_1 \right) \frac{\Psi'(v_1)}{p_2} \left[ \frac{\partial q_1}{\partial \phi_1} \frac{\partial^2 \phi}{\partial \phi \partial \phi_1} + \frac{\partial q_2}{\partial \phi_1} \frac{\partial^2 \phi}{\partial \phi \partial \phi_2} + \frac{\partial q_2}{\partial \phi_2} \frac{\partial^2 \phi}{\partial \phi \partial \phi_1} \right]$$

Substituting $p_2$ by its value in the equation (18), we obtain equation (26).

3 - Proof of proposition 4

In this appendix, we compute the optimal quality levels and access charge under asymmetric information. First, we determine the optimal quality levels. We need to differentiate the Hamiltonian of the optimization program with respect to quality variables $v_1$ and $v_2$. We have the following equations:

$$\psi'(v_1) = p_1 \frac{\partial q_1}{\partial \phi_1} + p_2 \frac{\partial q_2}{\partial \phi_1} - \frac{\partial q_2}{\partial \phi_1} \frac{\partial C_2}{\partial \phi_2} - \frac{\partial Q}{\partial \phi_1} \frac{\partial C_0}{\partial \phi} - \frac{\partial q_1}{\partial \phi_1} \frac{\partial C_1}{\partial \phi}$$

$$\psi'(v_2) = (1 + \lambda) \left[ p_1 \frac{\partial q_1}{\partial \phi_2} + p_2 \frac{\partial q_2}{\partial \phi_2} - \frac{\partial q_2}{\partial \phi_2} \frac{\partial C_2}{\partial \phi_1} - \frac{\partial Q}{\partial \phi_2} \frac{\partial C_0}{\partial \phi} - \frac{\partial q_1}{\partial \phi_2} \frac{\partial C_1}{\partial \phi} \right]$$

This implies the following equation system:

$$\begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \frac{\partial q_2}{\partial p_2} \\ \frac{\partial q_1}{\partial \phi} & \frac{\partial q_2}{\partial \phi} \\ \frac{\partial q_1}{\partial \phi_1} & \frac{\partial q_2}{\partial \phi_2} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = - \frac{\lambda}{\lambda + 1} F \left( v_1 \right) \left[ \psi'(v_1) \frac{\partial q_1}{\partial \phi_1} + \psi'(v_1) \frac{\partial q_2}{\partial \phi_1} \right]$$

$$+ \frac{\lambda}{\lambda + 1} F \left( v_1 \right) \left[ \psi'(v_1) \frac{\partial q_1}{\partial \phi_2} + \psi'(v_1) \frac{\partial q_2}{\partial \phi_2} \right]$$

$$+ \frac{\lambda}{\lambda + 1} F \left( v_1 \right) \left[ \psi'(v_1) \frac{\partial q_1}{\partial \phi_1} + \psi'(v_1) \frac{\partial q_2}{\partial \phi_2} \right]$$

$$+ \frac{\lambda}{\lambda + 1} F \left( v_1 \right) \left[ \psi'(v_1) \frac{\partial q_1}{\partial \phi_2} + \psi'(v_1) \frac{\partial q_2}{\partial \phi_2} \right]$$

$$+ \frac{\lambda}{\lambda + 1} F \left( v_1 \right) \left[ \psi'(v_1) \frac{\partial q_1}{\partial \phi_1} + \psi'(v_1) \frac{\partial q_2}{\partial \phi_2} \right]$$
Using Cramer rule, we solve this system and we obtain the following equations:

\[
p_1 = C_1^1 q_1 + C_1^0 \frac{\partial Q}{\partial q_1} \frac{\partial q_2}{\partial q_1} \frac{2\psi'(v_1)}{\partial q_2} \frac{\partial^2 \phi}{\partial \lambda \partial \psi} + \frac{\psi'(v_2)}{\partial q_2} \frac{\partial q_1}{\partial q_1}
\]

\[
+ \frac{\lambda}{\lambda + 1} F \begin{pmatrix}
\frac{\partial^2 \phi}{\partial \lambda \partial \psi} + \frac{\psi'(v_1)}{\partial q_1} \frac{\partial q_1}{\partial \lambda}
\end{pmatrix}
\]

And:

\[
p_2 = C_2^2 q_2 + C_2^0 \frac{\partial Q}{\partial q_2} \frac{\partial q_1}{\partial q_2} \frac{2\psi'(v_1)}{\partial q_1} \frac{\partial^2 \phi}{\partial \lambda \partial \psi} + \frac{\psi'(v_2)}{\partial q_2} \frac{\partial q_1}{\partial q_1}
\]

\[
+ \frac{\lambda}{\lambda + 1} F \begin{pmatrix}
\frac{\psi'(v_1)}{\partial q_1} \frac{\partial q_1}{\partial \lambda + 1} \frac{\partial^2 \phi}{\partial \lambda \partial \psi} - \frac{\psi'(v_1)}{\partial q_1} \frac{\partial q_1}{\partial \lambda}
\end{pmatrix}
\]

Since \( \frac{\partial^2 \phi}{\partial \lambda \partial \psi} = 0 \), these two equations become:

\[
p_1 = C_1^1 q_1 + C_1^0 \frac{\partial Q}{\partial q_1} \frac{\partial q_2}{\partial q_1} \frac{2\psi'(v_1)}{\partial q_2} \frac{\partial^2 \phi}{\partial \lambda \partial \psi}
\]

\[
+ \frac{\lambda}{\lambda + 1} F \begin{pmatrix}
\frac{\psi'(v_1)}{\partial q_1} \frac{\partial^2 \phi}{\partial \lambda \partial \psi} + \frac{\psi'(v_1)}{\partial q_1} \frac{\partial q_1}{\partial \lambda}
\end{pmatrix}
\]

\[
p_2 = C_2^2 q_2 + C_2^0 \frac{\partial Q}{\partial q_2} \frac{\partial q_1}{\partial q_2} \frac{2\psi'(v_1)}{\partial q_1} \frac{\partial^2 \phi}{\partial \lambda \partial \psi} + \frac{\psi'(v_2)}{\partial q_2} \frac{\partial q_1}{\partial q_1}
\]

\[
+ \frac{\lambda}{\lambda + 1} F \begin{pmatrix}
-\frac{\psi'(v_1)}{\partial q_1} \frac{\partial q_1}{\partial \lambda} \frac{\partial^2 \phi}{\partial \lambda \partial \psi}
\end{pmatrix}
\]

Next, we determine the optimal access charge as a function of quality levels, under asymmetric information. Substituting \( p_2 \) by its value given in (26), we obtain:
\[ a'^{l} = \frac{C_{l}^{0}}{q_{l}} \left[ 1 + \frac{\lambda}{\lambda + 1 q_{l}} \right] \]

\[ + \frac{\lambda}{\lambda + 1 q_{l}} \left[ C_{l}^{2} \frac{2\psi'(v_{1})}{\partial y_{2}} \frac{\partial y_{1}}{\partial y_{2}} \frac{\partial y_{1}}{\partial y_{2}} + \frac{\psi'(v_{2})}{\partial y_{1}} \frac{\partial y_{2}}{\partial y_{2}} \frac{\partial y_{1}}{\partial y_{1}} \right] \]

\[ + \frac{\lambda}{\lambda + 1 q_{l}} \left[ \frac{\lambda}{\lambda + 1 f} \frac{\psi'(v_{1})}{\partial y_{2}} \frac{\partial y_{1}}{\partial y_{2}} \frac{\partial y_{1}}{\partial y_{2}} \frac{\partial y_{1}}{\partial y_{2}} \right] \]

If we set:

\[ I(\psi''(v_{1}), \frac{\partial \phi_{1}}{\partial \theta}, q_{1_{1}}, q_{2_{1}}, q_{1_{2}}, q_{2_{2}}) = \frac{\lambda}{\lambda + 1 q_{l}} \left[ \frac{\lambda}{\lambda + 1 f} \frac{\psi'(v_{1})}{\partial y_{2}} \frac{\partial y_{1}}{\partial y_{2}} \frac{\partial y_{1}}{\partial y_{2}} \right] \]

We obtain:

\[ a' = a^{c} + I(\psi''(v_{1}), \frac{\partial \phi_{1}}{\partial \theta}, q_{1_{1}}, q_{2_{1}}, q_{1_{2}}, q_{2_{2}}) \]

**References**


