EXPANDED MIYAZAWA FRAMEWORK: LABOR AND CAPITAL INCOME, SAVINGS, CONSUMPTION, AND INVESTMENT LINKS

by

Michael Sonis and Geoffrey J.D. Hewings

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Expanded Miyazawa Framework: Labor and Capital Income, Savings, Consumption, and Investment Links.

Michael Sonis
Regional Economics applications Laboratory, University of Illinois, Urbana, IL 61801, USA and Bar Ilan University, Ramat Gan, 54700 Israel. e-mail:sonism@mail.biu.ac.il

Geoffrey J.D. Hewings
Regional Economics applications Laboratory, University of Illinois, Urbana, IL 61801, USA and Bar Ilan University, Ramat Gan, Israel. e-mail: hewings@uiuc.edu

1. Introduction

In two seminal publications of Miyazawa (1960, 1968), summarized in Miyazawa (1976), the author developed a generalization of the Keynesian income multiplier in the form of a matrix of inter-relational income multipliers.

The matrix Miyazawa model has a form:

$$\begin{pmatrix} x \\ Y \end{pmatrix} = \begin{pmatrix} A & C \\ V & 0 \end{pmatrix} \begin{pmatrix} x \\ Y \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix} \tag{1}$$

Here A is a matrix of direct input coefficients; x is the gross output, f is the final demand, the vector Y represents total income,[1] the matrix V represents the value-added ratios of households; the vector g represents the exogenous income; the matrix C represents the consumption expenditures.

The following analysis holds, (see Hewings *et al.* (1999), pp. 22-24) Miyazawa considered the following 2x2 block-matrix

$$M = \begin{pmatrix} A & C \\ V & 0 \end{pmatrix} \tag{2}$$

[1] In Pyatt (1998), discussion is raised of the distinction between factor income (Miyazawa) and institutional income (social accounting models) in more extensive input-output/social accounting systems. It is not clear in the Miyazawa system what types of other income are included in g in equation (1), referred to as exogenous income. In all probability, it will be less inclusive that the institutional income found in the social accounting system.

The Leontief inverse for the Miyazawa matrix (2) has a form:

$$B(M)=(I-M)^{-1}=$$
$$=\begin{pmatrix} I & BC \\ 0 & I \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & K \end{pmatrix}\begin{pmatrix} B & 0 \\ VB & I \end{pmatrix}=\begin{pmatrix} B+BCKVB & BCK \\ KVB & K \end{pmatrix}= \tag{3}$$
$$=\begin{pmatrix} I & 0 \\ V & I \end{pmatrix}\begin{pmatrix} \Delta & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} I & C \\ 0 & I \end{pmatrix}=\begin{pmatrix} \Delta & \Delta C \\ VB & I+V\Delta C \end{pmatrix}$$

where $B=(I-A)^{-1}$ is the Leontief interindustrial inverse matrix, $L = VBC$ is the matrix of inter-income groups coefficients, and:

$$K=(I-L)^{-1}=(I-VBC)^{-1}=I+V\Delta C \tag{4}$$

is the Miyazawa interrelational income multiplier or generalized Keynesian multiplier, and:

$$\Delta=(I-A-CV)^{-1}=B+BCKVB \tag{5}$$

is an enlarged Leontief inverse.

Also the Miyazawa fundamental equations of income formation from capital hold:

$$\begin{cases} V\Delta = KVB \\ \Delta C = BCK \end{cases} \tag{6}$$

In this paper we extend the Miyazawa scheme of matrix inter-relational income multiplier by incorporating in it the labor and capital income, savings, consumption and investments. Applications to the construction of Sraffa-Leontief-Miyazawa model and to the Pyatt-Round Social Accounting Model are considered in detail.

2. Extension considering Wages, Savings, Consumption and Investment

The concept of extension presented in this paper is stimulated by the matrix presentation of Sonis and Hewings (1988) of what may be referred to as an ‘onion-skin” approach to demographic-economic impact analysis (see Stone, 1981; Batey and Madden, 1983; and Batey, 1985). Such an extension can be achieved by consideration of the block-matrices $V=\begin{pmatrix} W \\ S \end{pmatrix}$ and $C=\begin{pmatrix} \tilde{C} & \tilde{I} \end{pmatrix}$

where the matrix W is the matrix of wages, the matrix S is the matrix of the savings coefficients, the matrix $\tilde{C}$ is the matrix of households consumption and the matrix $\tilde{I}$ is the matrix of investment coefficients.

In this case the Miyazawa block-matrix (2) obtains the form of 3x3 block-matrix:

$$M = \begin{pmatrix} A & \tilde{C} & \tilde{I} \\ W & 0 & 0 \\ S & 0 & 0 \end{pmatrix} \tag{7}$$

The formulae (3)-(6), thus, imply that the Miyazawa matrix of inter-income groups coefficients multiplier obtains the form:

$$L = \begin{pmatrix} W \\ S \end{pmatrix} B \begin{pmatrix} \tilde{C} & \tilde{I} \end{pmatrix} = \begin{pmatrix} WB\tilde{C} & WB\tilde{I} \\ SB\tilde{C} & SB\tilde{I} \end{pmatrix} \tag{8}$$

and the Miyazawa inter-relational income multiplier or generalized Keynesian multiplier is (see Sonis and Hewings, 2000a):

$$\begin{aligned} K &= (I-L)^{-1} = \left(I - \begin{pmatrix} W \\ S \end{pmatrix} B \begin{pmatrix} \tilde{C} & \tilde{I} \end{pmatrix} \right)^{-1} = \\ &= I + \begin{pmatrix} W \\ S \end{pmatrix} \Delta \begin{pmatrix} \tilde{C} & \tilde{I} \end{pmatrix} = \begin{pmatrix} I + W\Delta\tilde{C} & W\Delta\tilde{I} \\ S\Delta\tilde{C} & S\Delta\tilde{I} \end{pmatrix} \end{aligned} \tag{9}$$

and the enlarged Leontief inverse is:

$$\begin{aligned} \Delta &= \left(I - A - \begin{pmatrix} \tilde{C} & \tilde{I} \end{pmatrix} \begin{pmatrix} W \\ S \end{pmatrix} \right) = \left(I - A - \tilde{C}W - \tilde{I}S \right)^{-1} = \\ &= B + B\begin{pmatrix} \tilde{C} & \tilde{I} \end{pmatrix} K \begin{pmatrix} W \\ S \end{pmatrix} B \end{aligned} \tag{10}$$

Also the Miyazawa fundamental equations of income formation hold:

$$\begin{cases} \begin{pmatrix} W \\ S \end{pmatrix} \Delta = K \begin{pmatrix} W \\ S \end{pmatrix} B \\ \Delta \begin{pmatrix} \tilde{C} & \tilde{I} \end{pmatrix} = B \begin{pmatrix} \tilde{C} & \tilde{I} \end{pmatrix} K \end{cases} \tag{11}$$

3. Extensions Linking Capital and Labor

Introducing the division of total income, *Y*, into wages of labor and profits of capital with the help of a series of block-column and block-row vectors. The first block-column vector, $Y = \begin{pmatrix} y_K \\ y_L \end{pmatrix}$, represents total income, the second, $V = \begin{pmatrix} V_K \\ V_L \end{pmatrix}$, with the components V_K, V_L, represents the income from capital profits and labor wages, the block-column vector $g = \begin{pmatrix} g_K \\ g_L \end{pmatrix}$ represents the exogenous income from capital and labor, and the block-row vector $C = (C_K \quad C_L)$ represents the consumption expenditures from wages and profits. We will obtain from the Miyazawa scheme (1) - (2) the following model (Kimura *et. al.*,2000):

$$\begin{pmatrix} x \\ y_K \\ y_L \end{pmatrix} = \begin{pmatrix} A & C_K & C_L \\ V_K & 0 & 0 \\ V_L & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y_K \\ y_L \end{pmatrix} + \begin{pmatrix} f \\ g_K \\ g_L \end{pmatrix} \tag{12}$$

The model (12) includes two building blocks representing separately the income generation from capital and labor:

$$\begin{pmatrix} x \\ y_K \end{pmatrix} = \begin{pmatrix} A & C_K \\ V_K & 0 \end{pmatrix} \begin{pmatrix} x \\ y_K \end{pmatrix} + \begin{pmatrix} f \\ g_K \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} x \\ y_L \end{pmatrix} = \begin{pmatrix} A & C_K \\ V_L & 0 \end{pmatrix} \begin{pmatrix} x \\ y_L \end{pmatrix} + \begin{pmatrix} f \\ g_L \end{pmatrix} \tag{14}$$

These building blocks have the original Miyazawa form of income generation. Therefore, the following analysis holds (see Hewings *et al.* 1999, pp. 22-24).

Consider, at first, the Miyazawa model (13) for income generation connected with capital. Miyazawa evaluated the following block matrix

$$M_K = \begin{pmatrix} A & C_K \\ V & 0 \end{pmatrix} \tag{15}$$

The Leontief inverse for the Miyazawa matrix (15) has a form:

$$B(M_K)=(I-M_K)^{-1}=$$
$$=\begin{pmatrix} I & BC_K \\ 0 & I \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & K_K \end{pmatrix}\begin{pmatrix} B & 0 \\ V_K B & I \end{pmatrix}=\begin{pmatrix} B+BC_K K_K V_K B & BC_K K_K \\ K_K V_K B & K_K \end{pmatrix}=$$
$$=\begin{pmatrix} I & 0 \\ V_K & I \end{pmatrix}\begin{pmatrix} \Delta_K & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} I & C_K \\ 0 & I \end{pmatrix}=\begin{pmatrix} \Delta_K & \Delta_K C_K \\ V_K B & I+V_K \Delta_K C_K \end{pmatrix} \quad (16)$$

where $B=(I-A)^{-1}$ is the Leontief inter-industrial inverse matrix and,

$$K_K=(I-V_K BC_K)^{-1}=I+V_K\Delta_K C_K \quad (17)$$

is the Miyazawa capital inter-relational income multiplier or generalized Keynesian multiplier, and

$$\Delta_K=(I-A-C_K V_K)^{-1}=B+BC_K K_K V_K B \quad (18)$$

is an enlarged Leontief inverse.

Also the Miyazawa fundamental equations of income formation from capital hold:

$$\begin{cases} V_K\Delta_K=K_K V_K B \\ \Delta_K C_K=BC_K K_K \end{cases} \quad (19)$$

Analogously, it is possible to obtain similar results for the Miyazawa model (14) for income generation connected with labor, replacing the index *K* by the index *L* in formulae (16)-(19).

Returning to the Miyazawa model (12), we will present the results of the analysis using the substitutions into (15)-(19) of the block vectors $V=\begin{pmatrix} V_K \\ V_L \end{pmatrix}$ and $C=(C_K \quad C_L)$ instead of V_K, C_K (see Hewings *et al*, 1999, p. 24): for the Miyazawa matrix

$$M=\begin{pmatrix} A & C_K & C_L \\ V_K & 0 & 0 \\ V_L & 0 & 0 \end{pmatrix} \quad (20)$$

the Leontief inverse may be shown to have the form:

$$B(M)=(I-M)^{-1}=$$
$$=\begin{pmatrix} I & 0 & 0 \\ V_K & I & 0 \\ V_L & 0 & I \end{pmatrix}\begin{pmatrix} \Delta & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}\begin{pmatrix} I & C_K & C_L \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}= \qquad (21)$$
$$=\begin{pmatrix} \Delta & \Delta C_K & \Delta C_L \\ V_K\Delta & I+V_K\Delta C_K & V_K\Delta C_L \\ V_L\Delta & V_L\Delta C_K & I+V_L\Delta C_L \end{pmatrix}$$

where

$$\Delta=(I-A-CV)^{-1}=\left(I-A-\begin{pmatrix} C_K & C_L \end{pmatrix}\begin{pmatrix} V_K \\ V_L \end{pmatrix}\right)^{-1}=(I-A-C_K V_K-C_L V_L)^{-1} \qquad (22)$$

is the enlarged Leontief inverse the model (12), and

$$K=(I-VBC)^{-1}=\left(I-\begin{pmatrix} V_K \\ V_L \end{pmatrix}B\begin{pmatrix} C_K & C_L \end{pmatrix}\right)^{-1}=\left(I-\begin{pmatrix} V_K BC_K & V_K BC_L \\ V_L BC_K & V_L BC_L \end{pmatrix}\right)^{-1}=$$
$$=I+V\Delta C=\begin{pmatrix} I+V_K\Delta C_K & V_K\Delta C_L \\ V_L\Delta C_K & I+V_L\Delta C_L \end{pmatrix} \qquad (23)$$

is the Miyazawa labor-capital inter-relational income multiplier or generalized Keynesian multiplier for labor-capital.

The fine structure of the Miyazawa labor-capital inter-relational income multiplier, *K*, can be presented with the help of the interlaced inter-relational income multipliers

$$\Delta_{KLK}=(I-V_K\Delta_L C_K)^{-1}=I+V_K\Delta_L C_K;$$
$$\Delta_{LKL}=(I-V_L\Delta KC_L)^{-1}=I+V_L\Delta_K C_L \qquad (24)$$

This fine structure of the Miyazawa labor-capital inter-relational income multiplier, *K*, has a following form:

$$K=\begin{pmatrix} \Delta_{KLK} & K_K V_K BC_L\Delta_{LKL} \\ K_L V_L BC_K\Delta_{KLK} & \Delta_{LKL} \end{pmatrix} \qquad (25)$$

where the Miyazawa fundamental equations of income formation have a form:

$$\Delta C_K=\Delta_L C_K\Delta_{KLK};\ V_K\Delta=\Delta_{KLK}C_K K_K;$$
$$\Delta C_L=\Delta_K C_L\Delta_{LKL};\ V_L\Delta=\Delta_{LKL}C_L K_L \qquad (26)$$

Hence, the Leontief inverse (20) can be presented in the form:

$$B(M) = \begin{pmatrix} \Delta & \Delta_L C_K \Delta_{KLK} & \Delta_K C_L \Delta_{LKL} \\ \Delta_{KLK} V_K \Delta_K & \Delta_{KLK} & K_K V_K B C_L \Delta_{LKL} \\ \Delta_{LKL} V_L \Delta_L & K_L V_L B C_K \Delta_{KLK} & \Delta_{LKL} \end{pmatrix} \tag{27}$$

4. The Sraffa-Leontief-Miyazawa Income Distribution Model

The Sraffa-Leontief income distribution model can be presented in a form of Sonis, Hewings (2000b):

$$px = (1+r)pAx + w^* p(I-A)x \tag{28}$$

where x is a gross output, p is the vector of prices, A the matrix of direct input coefficients, f *is* the final demand, r is the uniform rate of capital profits and w^* is the wage rate of labor. Sraffa found the linear trade of relation between the rate of profits r and the wage rate w^*, (see Sraffa, 1960, p.22, and also Pasinetti, 1977, p. 115):

$$r = (1 - w^*)\frac{1-\mu}{\mu} \tag{29}$$

where μ is the Perronean maximal simple eigenvalue ($0 < \mu < 1$) corresponding to the left hand p and right hand x eigenvectors of positive matrix A:

$$pA = \mu p; \quad Ax = \mu x \tag{30}$$

Therefore, the capital income

$$V_K = rpA = r\mu p \tag{31}$$

and the labor income

$$V_L = w^* p(I - A) = w^*(1-\mu)p \tag{32}$$

generate the block-matrix of income

$$V = \begin{pmatrix} V_K \\ V_L \end{pmatrix} = \begin{pmatrix} r\mu p \\ w^*(1-\mu)p \end{pmatrix} = (1-\mu)\begin{pmatrix} 1-w^* \\ w^* \end{pmatrix} p \tag{33}$$

Next let us introduce the block-matrix of consumption $C = (C_K \quad C_L)$ whose components are the capital and labor consumption. With such specifications of block-matrices *V* and *C,* the Miyazawa model (12) with matrix (20) represents the Sraffa-Leontief-Miyazawa income generation model and formulae (21) -(28) represent its analysis.

5. Interconnection between the Miyazawa 3x3 Block-Matrix Model of Income Propagation and the Pyatt-Round Social Accounting Model

It is possible to prove the following:

Statement 1: The 3x3 block Pyatt-Round Social Accounting model SAM, (see details and references in Pyatt, Round (1985))

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} A_{11} & 0 & A_{13} \\ A_{21} & 0 & 0 \\ 0 & A_{32} & A_{33} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{34}$$

is equivalent to 3x3 block Miyazawa model:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} A_{11} & 0 & A_{13} \\ A_{21} & 0 & 0 \\ B_3 A_{32} A_{21} & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} x_1 \\ x_2 \\ x_\beta \end{pmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & A_{13} \\ A_{21} & 0 & 0 \\ B_3 A_{32} A_{21} & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_\beta \end{bmatrix} \tag{35}$$

where

$$B_3 = (I - A_{33})^{-1}; \; x_\beta = B_3 A_{32} x_2 + B_3 x_3 \tag{36}$$

The structure of the Pyatt-Round accounting multiplier M_1 for the system (34) and the structure of the Miyazawa multiplier M_2 for the system (35) can be ascertained with the help of an analytical technique developed for *nxn* block Leontief multipliers in the subsection 3 of Sonis and Hewings (2000a).

$$M_1 = \begin{pmatrix} I - A_{11} & 0 & -A_{13} \\ -A_{12} & I & 0 \\ 0 & -A_{32} & I - A_{33} \end{pmatrix}^{-1} =$$
$$= \begin{pmatrix} B_{11} & B_{11}A_{13}B_3A_{32} & B_{11}A_{13}B_3 \\ B_{22}A_{21}B_1 & B_{22} & B_{22}A_{21}B_1A_{13}B_3 \\ B_3A_{32}A_{21}B_{11} & B_3A_{32}B_{22} & B_{33} \end{pmatrix} \quad (37)$$

$$M_2 = \begin{pmatrix} I - A_{11} & 0 & -A_{13} \\ -A_{21} & I & 0 \\ -B_3A_{32}A_{21} & 0 & I \end{pmatrix}^{-1} = \begin{pmatrix} B_{11} & 0 & B_{11}A_{13} \\ B_{22}A_{21}B_1 & I & B_{22}A_{21}B_1A_{13}B_3 \\ B_3A_{32}A_{21}B_{11} & 0 & B_{33}^L \end{pmatrix} \quad (38)$$

where

$$\begin{aligned} B_1 &= (I - A_{11})^{-1}; \\ B_{11} &= (I - A_{11} - A_{13}B_3A_{32}A_{21})^{-1}; \\ B_{22} &= (I - A_{21}B_1A_{13}B_3A_{32})^{-1}; \\ B_{33}^L &= (I - B_3A_{32}A_{21}B_1A_{13})^{-1}; \\ B_{33} &= (I - A_{33} - A_{32}A_{21}B_1A_{13})^{-1} = B_{33}^L B_3 \end{aligned} \quad (39)$$

Further, it is possible to compare the SAM with original 2x2 Miyazawa system. First of all, we consider a statement, which is implicitly stated in paper of Pyatt (1998).

Statement 2: The 3x3 block SAM model (34) is equivalent to the pair of 2x2 block Miyazawa models $(\alpha),(\beta)$:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{13}B_3A_{32} \\ A_{21} & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} x_\alpha \\ x_2 \end{pmatrix} \quad (\alpha)$$

where $x_\alpha = x_1 + A_{13}B_3x_3$ and

$$\begin{pmatrix} y_1 \\ y_3 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{13} \\ B_3A_{32}A_{21} & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_3 \end{pmatrix} + \begin{pmatrix} x_1 \\ x_\beta \end{pmatrix} \quad (\beta)\ (\beta)$$

where $x_\beta = B_3A_{32}x_2 + B_3x_3$

The Miyazawa multipliers for the systems $(\alpha),(\beta)$ are:

$$M_\alpha = \begin{pmatrix} I - A_{11} & -A_{13}B_3A_{32} \\ -A_{21} & I \end{pmatrix}^{-1} = \begin{pmatrix} B_{11} & B_{11}A_{13}B_3A_{32} \\ B_{22}A_{21}B_1 & B_{22} \end{pmatrix} \quad (40)$$

$$M_\beta = \begin{pmatrix} I - A_{11} & -A_{13} \\ -B_3A_{32}A_{21} & I \end{pmatrix}^{-1} = \begin{pmatrix} B_{11} & B_{11}A_{13} \\ B_3A_{32}A_{21}B_1 & B_{33}^L \end{pmatrix} \quad (41)$$

The analogous Statement 3 can be formulated:

Statement 3: The 3x3 block SAM model (34) is equivalent to the pair of 2x2 block models $(\gamma),(\delta)$,:

$$\begin{pmatrix} y_1 \\ y_3 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{13} \\ A_{32}A_{21} & A_{33} \end{pmatrix}\begin{pmatrix} y_1 \\ y_3 \end{pmatrix} + \begin{pmatrix} x_1 \\ x_\gamma \end{pmatrix} \quad (\gamma)$$

where $x_\gamma = A_{32}x_2 + x_3$ and

$$\begin{pmatrix} y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 & A_{21}B_1A_{13} \\ A_{32} & A_{33} \end{pmatrix}\begin{pmatrix} y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} x_\delta \\ x_3 \end{pmatrix} \quad (\delta)$$

where $x_\delta = A_{21}B_1x_1 + x_2$

The Schur-Miyazawa multipliers for the systems $(\gamma),(\delta)$ are, *cf*. subsection 2 from Sonis and Hewings (2000a):

$$M_\gamma = \begin{pmatrix} I - A_{11} & -A_{13} \\ -A_{32}A_{21} & I - A_{33} \end{pmatrix}^{-1} = \begin{pmatrix} B_{11} & B_{11}A_{13}B_3 \\ B_{33}A_{32}A_{21}B_1 & B_{33} \end{pmatrix} \quad (42)$$

$$M_\delta = \begin{pmatrix} I & -A_{21}B_1A_{13} \\ -A_{32} & I - A_{33} \end{pmatrix}^{-1} = \begin{pmatrix} B_{22} & B_{22}A_{21}B_1A_{13}B_3 \\ B_3A_{32}B_{22} & B_{33} \end{pmatrix} \quad (43)$$

Statements 1, 2, and 3 enable interpretations that are compatible with conventional economic reasoning. Moreover, the important fact is that the exclusion of the first column and row from the Pyatt-Round accounting multiplier, M_1, (see (40)) yeilds the Schur-Miyazawa multiplier M_δ; the exclusion of the second column and row gives the Schur-Miyazawa multiplier M_γ; and the exclusion of the third column and row gives the Miyazawa multiplier M_α.

In addition, the extensions of the of the SAM model through the introduction of different assets (including human capital) that provide factor services and including different types of institutions, lead to an *n*x*n* SAM model, whose accounting multiplier can be constructed by the analytical technique used for *n*x*n* Block Input-Output analysis (see Sonis and Hewings, 2000a).

6. Expansion by Layers of Wages, Profits, Savings, Consumption, Investment by Labor and Capital

This extension can be achieved by the consideration of following block-matrices:

$$V = \begin{pmatrix} W_L \\ S_L \\ P_K \\ S_K \end{pmatrix}; \; C = \begin{pmatrix} C_L & I_L & C_K & I_K \end{pmatrix} \tag{44}$$

where W_L, S_L, C_L, I_L are the wages, savings, consumption and investments from labor, P_K, S_K, C_K, I_K are the profits, savings, consumption and investments from capital. In this case, the Miyazawa matrix has the form:

$$M = \begin{pmatrix} A & C_L & I_L & C_K & I_K \\ W_L & 0 & 0 & 0 & 0 \\ S_L & 0 & 0 & 0 & 0 \\ P_K & 0 & 0 & 0 & 0 \\ S_K & 0 & 0 & 0 & 0 \end{pmatrix} \tag{45}$$

The Miyazawa matrix of inter-income groups coefficients multiplier obtains the form:

$$L = \begin{pmatrix} W_L \\ S_L \\ P_K \\ S_K \end{pmatrix} B \begin{pmatrix} C_L & I_L & C_K & I_K \end{pmatrix} = \begin{pmatrix} W_L BC_L & W_L BI_L & W_K BC_K & W_K BI_K \\ S_L BC_L & S_L BI_L & S_L BC_K & S_L BI_K \\ P_K BC_L & P_K BI_L & P_K BC_K & P_K BI_K \\ S_K BC_L & S_K BI_L & S_K BC_K & S_K BI_K \end{pmatrix} \tag{46}$$

and the Miyazawa inter-relational income multiplier or generalized Keynesian multiplier is:

$$K = (I - L)^{-1} = I + \begin{pmatrix} W_L \\ S_L \\ P_K \\ S_K \end{pmatrix} \Delta \begin{pmatrix} C_L & I_L & C_K & I_K \end{pmatrix} =$$

$$= \begin{pmatrix} I + W_L \Delta C_L & W_L \Delta I_L & W_K \Delta C_K & W_K \Delta I_K \\ S_L \Delta C_L & I + S_L \Delta I_L & S_L \Delta C_K & S_L \Delta I_K \\ P_K \Delta C_L & P_K \Delta I_L & I + P_K \Delta C_K & P_K \Delta I_K \\ S_K \Delta C_L & S_K \Delta I_L & S_K \Delta C_K & I + S_K \Delta I_K \end{pmatrix} \qquad (47)$$

with the enlarged Leontief inverse :

$$\Delta = \left(I - A - \begin{pmatrix} C_L & I_L & C_K & I_K \end{pmatrix} \begin{pmatrix} W_L \\ S_L \\ P_K \\ S_K \end{pmatrix} \right)^{-1} = \qquad (48)$$

$$\left(I - A - C_L W_L - I_L S_L - C_K P_K - I_K S_K \right) =$$

Also the Miyazawa fundamental equations of income formation hold:

$$\begin{cases} \begin{pmatrix} W_L \\ S_L \\ P_K \\ S_K \end{pmatrix} \Delta = K \begin{pmatrix} W_L \\ S_L \\ P_K \\ S_K \end{pmatrix} \\ \Delta \begin{pmatrix} C_L & I_L & C_K & I_K \end{pmatrix} = B \begin{pmatrix} C_L & I_L & C_K & I_K \end{pmatrix} \end{cases} \qquad (49)$$

The components of formulae (45) – (49) represent the different layers of the income formation and expenditure and savings and investment in the expended Miyazawa model.

7. Conclusion

This paper explores the possibility to enlarge the original Miyazawa 2x2-block income generation scheme to the case of many layers of labor and capital income, savings, consumption and investments. This “onion-skin” approach facilitates the inclusion of the Miyazawa income generation and propagation mechanism into the Sraffa-Leontief model and thus provides an explanation of the conceptual and structural interconnection of Miyazawa ideas with those independently elaborated within the Pyatt-Round Social Accounting scheme. Of course, as Pyatt

(1998) has noted, there are some important differences in the nature of the income propagation process in the Miyazawa and more traditional SAM systems, with the latter containing a more inclusive form of income (especially from non-wage and salary sources). However, these differences can still be handled within the general framework provided in this paper.

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