Taxation and the Diffusion of a Cleaner Technology

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REAL 01-T-1 March 2001

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First version: January 2000

This version: March 2001

Abstract

A regulator faces an environmental problem because of the polluting activity of firms. The latter can adopt a new and less polluting production technology by spending an actualized investment cost decreasing exponentially with the adoption date. When firms adopt the cleaner technology, they produce more, pollute less, pay fewer emission taxes and as consequences have greater profit and the social welfare is higher. If the cost of the immediate adoption of the cleaner technology is relatively high and the environmental taxation scheme is well designed, firms will adopt it at finite but different dates even though the model is symmetric and there is no informational asymmetry. Moreover, we show that technological diffusion is socially optimal. The social adoption date of the first innovator is earlier than the private one whereas, the contrary occurs with the second innovator. Subsidies may be used to induce the socially-optimal adoption dates.

Keywords: Environment; Taxation; Technological diffusion.

JEL classification: D62; H32; O31

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1. Introduction

This paper tries to answer the question of how to induce firms to adopt less polluting technologies, to characterize this adoption process and compare it to the socially optimal one. Our solution consists of introducing an adequate environmental tax.

Milliman and Prince (1989) have evaluated the incentive effects of five environmental policy instruments (emission taxes, subsidies, auctioned permits, issued marketable permits and performance standards) to promote the development and adoption of advanced pollution abatement technology. They support the view that taxes and auctioned permits are the most effective policy instruments. Jung, Krutilla and Boyd (1996) have extended this comparative approach to the industry level.

Farzin and Kort (2000) examine the effect of a higher pollution tax rate on abatement investment, both under full certainty and when the timing or the size of the tax increase is uncertain. They show the possibility that a higher pollution tax rate induces more pollution and that a credible threat to accelerate the tax increase can lead to a more abatement investment. Stranlund (1997) considers public aid to encourage the adoption of superior emission-control technologies combined with monitoring. This strategy is interesting when monitoring is difficult because the sources of pollution are widely dispersed or when emissions are not easily measured as in non-point pollution problems. Technological aid reduces the direct enforcement effort necessary for firms to reach the compliance goal. Consequently, firms adopt better technologies, which serve to promote further innovative activity.

To bridge the gap between the private switching time and the socially desirable one, Dosi and Moretto (1997) recommend that regulators should focus on the sources of technological inertia so as to increase the private opportunity cost of postponing potentially-profitable-environmental innovations. Dosi and Moretto (2000) examine the implications of the sources of inertia on the design of public incentives aimed at accelerating adoption of cleaner technologies when the policy-maker is imperfectly informed on the private switching costs.

The diffusion of a new technology has been analyzed by Reinganum (1981). She considers an industry composed of two firms which can adopt a cost reducing technology within a time t. She shows that even in the case of identical firms and complete information there is diffusion of innovation over time because one firm innovates before the other and gains more. She has assumed that the payoff functions of firms are globally concave in their arguments. Besides, these functions are not differentiable when adoption is simultaneous (i.e. at $T_1 = T_2$, see Reinganum (1981) page 397). Fudenberg and Tirole (1985) make less strong condition on the payoffs of firms to obtain quasiconcavity. They show that under certain conditions, there is diffusion whereas under others, firms adopt this new technology simultaneously. Hoppe (2000) extends the work of Fudenberg and Tirole to include uncertainty regarding the profitability of a new technology. She shows that there may be second-mover advantages because of informational spillovers. Dutta et al. (1995) get a similar result in a context where the later innovator continues to develop the technology and eventually markets a higher-quality good.

The impact of adoption timing on social welfare has been studied by some authors. Riordan (1992) shows that price and entry regulations, in many cases, beneficially slow down technology adoption and, in some other cases, change the order in which firms adopt new technologies by speeding up one firm's adoption date and slowing down the other's. Stoneman and Diederen (1994) conjecture that 'diffusion policy should not proceed upon a presumption that faster is always better' (p.929). A partial welfare analysis, made by Hoppe, reveals several market failures[1] and suggests that policy intervention should adequately depend on the nature of uncertainty and the rate of technological progress.

This paper is an extension of the work of Carraro and Topa (1991). It differs from the existing literature mainly by the fact that we study diffusion within a framework where firms adopt the new technology for reducing pollution. Moreover, we

[1] She prove normative conclusions (Propositions 4 and 5) about sufficiently small changes of the equilibrium adoption dates, which show that the equilibrium adoption behavior tends to diverge from the social optimum.

completely characterize the socially optimal adoption process and compare it to the private one.

The most important differences of this model from that of Carraro and Topa are the function $\rho(t)$, which represents the actualized cost of adopting a less polluting technology by a firm at time t, and the intertemporal payoffs of firms and regulator. This function ρ makes the intertemporal objective functions of firms and regulator locally concave with respect to their arguments (i.e. supposes a weaker condition).

The symmetric model we consider consists of two firms located in a given country. Firms produce the same good sold on the market. A by-product of the production process is pollution. These firms can adopt a new, less polluting technology at date t by incurring an actualized investment cost $\rho(t)$ which decreases exponentially. In order to induce his firms to adopt the cleaner technology, the regulator taxes the emission of pollution. The regulator and firms maximize their intertemporal objective functions which take into account the cost of innovation. Our main interest is the case where the cost of immediate innovation (i.e. at date 0) is relatively high.

The regulator tries to induce firms to use the new technology because this implies greater social welfare. Indeed, firms produce more, pollute less, pay less environmental tax and have a higher profit than with the old (more polluting) technology. The regulator chooses the optimal tax parameter by maximizing the intertemporal social welfare. As a reaction, firms choose their optimal adoption dates by maximizing their intertemporal payoffs. We show that if the tax is well chosen, firms will adopt the cleaner innovation at different dates (diffusion). This is because each firm tries to have a competitive advantage by being the first innovator (which implies fewer emission taxes than the other non-innovating firm). We establish that technological diffusion is, in fact, socially optimal because the regulator prefers that the community supports only one investment cost in cleaner technology for a certain period until it decreases sufficiently. Though the social adoption date of the first innovator is earlier than the private one, the opposite happens for the second innovator[2]. Therefore, the regulator can give subsidies to firms so that they innovate at the socially appropriate dates.

[2] This important result is contrary to the one established by Carraro and Topa (Theorem 3).

The model is introduced in section 2. Section 3 analyzes the innovation game between firms given the taxation scheme introduced. Section 4 studies the regulator's optimal strategy and section 5 concludes.

2. The model

We consider two identical firms competing by quantities on a market where they offer the same good. The production process generates pollution (for example, sulfur dioxide SO_2). The regulator has decided to tax emission of pollution in order to protect the environment.

Before any environmental regulation is introduced, firms produce output using a single-product technology D characterized by a fixed emission / output ratio k. Thus, polluting emissions x_i are a linear function of firm i's output q_i: $x_i = k q_i$, k>0. If no environmental taxation is introduced, firms adopt technology D.

The marginal cost of production is c>0. No pollution abatement is possible with technology D: firms can reduce pollution only by reducing output. However, firms can adopt a new and more flexible technology F, characterized by abatement possibilities and a lower emission / output ratio.

The new technology F is a multiple-product technology that enables firms to produce an abatement good a_i jointly with output q_i. Firm i's residual emissions are $x_i = kq_i - a_i$. Hence, the new emission / output ratio k' is $k'_i = (kq_i - a_i) / q_i \leq k$. The unit abatement cost d' is set equal to d / k>0 (i.e. d=kd'>0). Because of this unit cost, pollution is not totally absorbed. Total abatement and emissions are $A = a_1 + a_2$ and $X = x_1 + x_2$, respectively.

Damages M(X) caused to the environment are a convex function of total emissions X :

$$M(X)=\lambda X^2$$

λ>0 increases with the sensitivity of consumers to the environment.

When the regulator introduces an emission tax, firms could be induced to invest in R&D in order to adopt the cleaner technology. In this case, each firm chooses the date at which the innovation will be available, and for each time period the abatement level a_i, and output q_i. Without loss of generality, we suppose that the regulator announces the emission tax parameter at time 0. Firms react by being engaged in the innovation game, in which each one decides whether to innovate or not, and at what date.

Each firm is asked to pay a tax t(X) per unit of residual emission which is positively correlated to total emissions. Therefore, t(X)=vX, where the parameter v>0 is chosen by the regulator, and the tax paid by firm i is $T_i(x_i, X) = vXx_i$.

The inverse demand function for output is[3] :

$$P(Q) = \alpha - \beta Q \quad \text{where } Q = q_1 + q_2, \alpha > c + 3d \text{ and } \beta > 0 \qquad (1)$$

Firms can adopt the new technology within a period t from the beginning of the research by spending an actualized monetary amount ρ(t). This investment cost could comprise the R&D cost and/or the cost of acquisition and installation of the new technology. Thus, we will use the terms innovation and adoption interchangeably. Function ρ is decreasing because of the existence of freely-available scientific research allowing firms to reduce the cost of innovation when they delay its adoption, and is convex because the innovation cost increases more rapidly as firms try to accelerate the date of adoption.

We model the cost of adopting the new technology at time t actualized at date 0 as[4] :

$$r(t) = be^{-mrt}, b > 0, m > 1, \text{ r > 0 is the discount rate} \qquad (2)$$

[3] The restriction α>c+3d is necessary for the tax parameter to be inferior to a positive value (see (A2) in Appendix 1).

[4] Carraro and Topa impose a strong condition on the function ρ making the intertemporal objective functions of firms globally concave with respect to their argument. This too strong condition is expressed in A.C (d) (page 19) and does not make it possible to prove the adoption by firms of the new, less polluting technology within a finite time unless the limits calculated (pages 20 and 33) are incorrect. They also assume, but do not prove, that firms adopt the new technology at different dates as shown by the expressions defining the intertemporal objective functions of firms and the regulator, which are not differentiable in $t_1 = t_2$ (page 19 and expression (13)). More precisely, they have ignored our expressions (11) and (17). Function ρ that we propose in this paper makes the intertemporal objective functions of firms and the regulator locally concave with respect to their arguments (i.e. we suppose a less strong condition).

For any v verifying conditions (A1) and (A2) (see Appendix 1), we need b, m and r be such as:

$$\frac{W_{FD}^t - W_{DD}^t}{mr} \leq b \tag{3}$$

where W_{FD}^t (resp. W_{DD}^t) is the instantaneous social welfare under an environmental tax and when only one firm has innovated (resp. no firm has innovated). Since the tax parameter v, verifying conditions (A1) and (A2), is minored and majored by strictly positive numbers, then $W_{FD}^t - W_{DD}^t$ (given by (A12) in Appendix 4) is majored by a strictly positive number independent of v. Therefore, by choosing mr sufficiently high, inequality (3) is feasible.

Inequality (3) means that the cost of instantaneous innovation (ρ(0)=b) is relatively high. For it to be fulfilled when we decrease r to zero, we increase m so that mr remains constant. In so doing, function ρ(t) does not change. The cost of innovation decreases more rapidly as m is greater.

Firms autonomously decide on their date of adoption of the cleaner technology at the beginning of the innovation game (date 0), and there is nothing in the model, such as informational externalities, that can incite them to change their strategy later (i.e. open-loop strategies).

If both firms use technology D, even in the presence of the emission tax , we have:

$$\Pi_i = \left[a - b(q_i + q_j)\right]q_i - (c + vkX)q_i = \left[a - (b + vk^2)(q_i + q_j) - c\right]q_i \tag{4}$$

$$i \neq j\ ,\ i\ ,\ j = 1\ ,\ 2$$

In the absence of the emission tax, the above expression is valid by setting v= 0.

If both firms adopt the new technology F after the introduction of the tax:

$$\begin{aligned}\Pi_i &= \left[a - b(q_i + q_j)\right]q_i - vX(kq_i - a_i) - cq_i - d / k.a_i \\ &= \left[a - (b + vk^2)(q_i + q_j) + vk(a_i + a_j) - c\right]q_i + \left[vk(q_i + q_j) - v(a_i + a_j) - d / k\right]a_i\end{aligned} \tag{5}$$

$$i \neq j\ ,\ i\ ,\ j = 1\ ,2$$

Finally, we consider the case in which one of the two firms (for example, firm 2) has innovated, whereas the other still produces using technology D:

$$\Pi_1 = \left[a - b(q_1 + q_2)\right]q_1 - cq_1 - vkXq_1 = \left[a - (b + vk^2)(q_1 + q_2) + vka_2 - c\right]q_1 \tag{6}$$

$$\Pi_2 = \left[a - b(q_1 + q_2)\right]q_2 - vX(kq_2 - a_2) - cq_2 - d / k.a_2$$
$$= \left[a - (b + vk^2)(q_1 + q_2) + vka_2 - c\right]q_2 + \left[vk(q_1 + q_2) - va_2 - d / k\right]a_2 \quad (7)$$

Following the background induction principle, first, we solve the game between firms in the second stage given the taxation scheme. Then, we analyze the behavior of the regulator which determines the optimal tax parameter.

3. The innovation game between firms

Given the tax imposed by the regulator in the first stage, firms engage in a dynamic game of innovation, deciding whether to adopt the cleaner technology or not, and if so at what date. Hence, we analyze the reaction of firms to a given tax parameter v.

Let f_{DD} and f^t_{DD} be the profits of firms when both use technology D respectively in the absence of emission tax and after the introduction of it. Let f^t_{FF} be the profit of a firm after both have innovated. Let f^t_{FD} be the profit of a firm when it has innovated while the other still uses technology D. Finally, f^t_{DF} is the profit of a firm when it still uses technology D, while the other has innovated. Total profits are denoted by Φ.

At each time period, firms decide their production levels (and abatement levels when they use the new technology) as the optimal strategies of a Nash-Cournot duopoly game (results are given in Appendix 1).

Conditions (A1) and (A2) in Appendix 1 enable us to rank all quantities, prices, emission / output ratios and profits in the four technological cases :

Output : $q_{DD} \geq q^t_{FD} > q^t_{FF} > q^t_{DF} = q^t_{DD} > 0 \qquad Q_{DD} > Q^t_{FF} > Q^t_{FD} > Q^t_{DD} > 0$

Price : $p^t_{DD} > p^t_{FD} > p^t_{FF} > p_{DD} > 0$

Abatement : $a^t_{FD} > a^t_{FF} > a^t_{DF} = a^t_{DD} = a_{DD} = 0 \qquad A^t_{FF} > A^t_{FD} > A^t_{DD} = A_{DD} = 0$

Emissions : $x_{DD} > x^t_{DD} = x^t_{DF} > x^t_{FF} > x^t_{FD} \geq 0 \qquad X_{DD} > X^t_{DD} > X^t_{FD} > X^t_{FF} > 0$

Emission / output ratios : $(x/q)_{DD} = (x/q)^t_{DD} = (x/q)^t_{DF} = k > (x/q)^t_{FF} > (x/q)^t_{FD} \geq 0$

Profits : $f_{DD} \geq f^t_{FD} > f^t_{FF} > f^t_{DF} = f^t_{DD} > 0 \qquad \Phi_{DD} > \Phi^t_{FD} > \Phi^t_{FF} > \Phi^t_{DD} > 0$

Output is highest without taxation (DD), and is lowest when the regulator tax emissions, whereas firms still use technology D. Total output Q rises as innovation spreads within the industry, because the impact of tax emissions is less serious.

Total emissions are lowest in the (FF/t) case, implying that environmental innovation enables the regulator to achieve a lower emission level than with the old technology.

The profit squeeze induced by the emission tax is much lower when firms adopt the new technology.

Therefore, in the presence of a suitable tax, firms produce more, pollute less, pay lower pollution tax and have a greater profit when they use the cleaner technology.

In order to understand the innovation game in the industry, let us analyze the case in which only one firm innovates (the FD/t case). Notice that the firm that innovates first, will gain substantially from innovation by exploiting the fact that the other has to reduce production for limiting the burden of the emission tax. Residual emissions and the emission/output ratio are lower than in all other cases for the firm that adopts first. As a result, production q_{FD}^{t} is greater than in all other cases and the profit f_{FD}^{t} is very high, thus, making the industry's profit Φ_{FD}^{t} larger than in the (FF/t) case, even if the profit of the non-innovating firm remains at the $f_{DF}^{t} = f_{DD}^{t}$ level. All these reasons incite each firm to be the first innovator, but they must be compared to the cost of early innovating.

If t_1 and t_2 are the adoption dates of firm 1 and 2, respectively, then firm 1's intertemporal objective function is :

$$V_1(t_1,t_2) = \begin{cases} g_1^1(t_1,t_2) & if\ t_1 < t_2 \\ g_1^2(t_1,t_2) & if\ t_1 > t_2 \\ g(t) & if\ t_1 = t_2 \end{cases} \tag{8}$$

where,

$$g_1^1(t_1,t_2) = \int_0^{t_1} f_{DD}^{t} e^{-rt}\, dt + \int_{t_1}^{t_2} f_{FD}^{t} e^{-rt}\, dt + \int_{t_2}^{+\infty} f_{FF}^{t} e^{-rt}\, dt - r(t_1) \tag{9}$$

$$g_1^2(t_1,t_2) = \int_0^{t_2} f_{DD}^{t} e^{-rt}\, dt + \int_{t_2}^{t_1} f_{DF}^{t} e^{-rt}\, dt + \int_{t_1}^{+\infty} f_{FF}^{t} e^{-rt}\, dt - r(t_1) \tag{10}$$

$$g(t) = \int_0^{t} f_{DD}^{t} e^{-rt}\, dt + \int_{t}^{+\infty} f_{FF}^{t} e^{-rt}\, dt - r(t) \tag{11}$$

Firm 1's payoff is $g_1^1(t_1, t_2)$ if it innovates first and $g_1^2(t_1, t_2)$ if firm 2 innovates first. The payoff of each one is g(τ) if they decide to innovate simultaneously at $t = t_1 = t_2$. Notice that $t_i = +\infty$ means that firm i never innovates.

Theorem 1. *Suppose (A1),(A2) and v£2l, then there exist two Nash equilibria of the innovation game between firms :*

$$(t_1^*, t_2^*) = (\hat{t}, \bar{t}) \text{ and } (t_1^*, t_2^*) = (\bar{t}, \hat{t}),\ 0 < \hat{t} < \bar{t} < +\infty$$

These optimal adoption dates are accelerated by a higher taxation parameter.

Proof. See Appendix 2.

Even if the model is symmetric and there is no uncertainty, the innovation game is characterized by diffusion in adoption dates. The intuition behind this result is the following : Each firm has an incentive to innovate first with respect to the cases of simultaneous innovation or no innovation since $f_{FD}^t > f_{FF}^t > f_{DF}^t = f_{DD}^t$. Indeed, the first adopter has a lower emission / output ratio enabling it to produce more, pollute less and consequently pays fewer emission taxes. In addition, it exploits the fact that the non-innovating firm must produce less to avoid important pollution taxes. However, the first will support higher R&D cost. Therefore, the first innovator has to compare the competitive advantage of being first to the higher R&D cost. This comparison shows that it is profitable to adopt this cleaner technology first.

In the following, the adoption dates will be t_1 and t_2 where the subscripts 1 and 2 indicate respectively the firm that adopts first and second, rather than the identity of the firm.

4. The regulator's optimal strategy

In this section, using the Nash-perfect equilibrium concept, we derive the regulator's optimal strategy which consists of the introduction of a suitable taxation scheme that may induce firms to adopt the new technology.

By maximizing his inter-temporal social welfare, the regulator decides whether to set a positive tax parameter or not, and if so, the optimal tax parameter.

The regulator's social welfare at time t is the sum of the consumer welfare and firms profits:

$$W = CS + \Phi \tag{12}$$

Consumer welfare is the sum of consumer surplus derived from the consumption of Q and of pollution taxes, minus damages from pollution :

$$CS = \int_0^{Q^*} p(Q)\,dQ - p(Q^*)Q^* + t(X^*)X^* - M(X^*) \tag{13}$$

where Q^* and X^* are the optimal values of total output and emissions calculated in stage two of the game.

Therefore, we have[5] :

$$W = \int_0^{Q^*} p(Q)dQ - cQ^* - \frac{d}{k}A^* - M(X^*) \tag{14}$$

Notice that taxes do not appear in the above expression because they are pure transfers from firms to consumers.

The socially optimal total output, abatement and emissions are[6] :

$$\overline{Q} = \frac{a-c-d}{b} \quad , \quad \overline{A} = \frac{2lk^2(a-c)-(b+2lk^2)d}{2blk} \quad , \quad \overline{X} = k\overline{Q} - \overline{A} = \frac{d}{2lk}$$

It is easy to verify that the socially optimal total abatement increases with λ, whereas emissions decrease with λ.

The above quantities represent the Pareto optimum with which we will compare the equilibrium quantities resulting from the implementation of the environmental policy.

Using the following notations, we compute the consumer surplus and social welfare in Appendix 3 :

$$W(t=0) = W_{DD}\ ,\ W(DD/t) = W_{DD}^t\ ,\ W(FF/t) = W_{FF}^t\ ,\ W(FD/t) = W_{FD}^t = W_{DF}^t$$

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[5] This general formulation includes the (DD/t) case in which the equilibrium abatement level A^* is zero.

[6] $\overline{A} \geq 0$ *iff* $l \geq \dfrac{bd}{2k^2(a-c-d)}$. Second order conditions are satisfied.

Proposition 1. *Suppose (A1),(A2) and v£2l, then :*

$$W_{FF}^{t} > W_{FD}^{t} > W_{DD}^{t}$$

Proof. See Appendix 4.

Proposition 1 shows that the adoption of environmental innovation increases the social welfare because the cleaner technology enables firms to produce more with less pollution.

We denote the intertemporal social welfare by :

$$\begin{cases} W^{t} \ \text{if the regulator imposes a tax} \\ W^{0} \ \text{if no taxation is imposed} \end{cases} \tag{15}$$

where,

$$W^{t} = \begin{cases} W^{t}(t_1, t_2) = \int_0^{t_1} W_{DD}^{t} e^{-rt}\, dt + \int_{t_1}^{t_2} W_{FD}^{t} e^{-rt}\, dt + \int_{t_2}^{+\infty} W_{FF}^{t} e^{-rt}\, dt - r(t_1) - r(t_2),\ t_1 < t_2 & (16) \\ W^{t}(t) = \int_0^{t} W_{DD}^{t} e^{-rt}\, dt + \int_{t}^{+\infty} W_{FF}^{t} e^{-rt}\, dt - 2r(t),\ t_1 = t_2 = t & (17) \end{cases}$$

$$W^{0} = \int_0^{+\infty} W_{DD} e^{-rt}\, dt \tag{18}$$

We recall that t_1 and t_2 are the adoption dates of the first and second innovator, respectively.

Theorem 2. *Suppose that the discount rate is sufficiently close to zero and* $l \in \left]l_1, l_{A2}\right]$*, then the optimal tax parameter is* $\bar{v} = \frac{4}{3} l$.

Proof. See Appendix 5.

Hence, the regulator finds it optimal to tax emissions if and only if consumers' valuation of environmental quality is sufficiently high. The upper bound l_{A2} guarantees non-negative residual emissions. When λ belongs to $\left]l_1, l_{A2}\right]$, conditions (A1) and (A2) are verified by $\bar{v}$.

Proposition 2. *The optimal emission tax parameter determines an inefficient allocation in terms of total output, but enables the regulator to achieve the socially-optimal level of total pollution, i.e.:*

$$Q_{FF}^{t}(\overline{v}) < \overline{Q} \quad , \quad X_{FF}^{t}(\overline{v}) = \overline{X}$$

Proof. Immediate.

In the following, we will compare the socially optimal innovation process to the private one. Let t_{is} be the socially optimal adoption dates.

Theorem 3. *Suppose (A1),(A2) and v £ 2l, then there exists a unique pair* (t_{1s}, t_{2s}) *that maximizes intertemporal social welfare, verifying:*

$$0 \leq t_{1s} < t_{2s} < +\infty$$

Proof. See Appendix 6.

There is an incentive for the regulator that firms innovate simultaneously because the social welfare is greater than when only one firm innovates. But the positive difference $W_{FF}^{t} - W_{FD}^{t}$ is not high enough, and that is why the regulator prefers to support only one innovation cost earlier and wait until the second cost of innovation decreases sufficiently. Therefore, diffusion in adoption dates, which has been shown to be optimal for firms, is actually socially optimal.

Theorem 4. *Suppose (A1),(A2) and v £ 2l, then we have:*

$$0 \leq t_{1s} < t_{1}^{*} < t_{2}^{*} < t_{2s} < +\infty$$

Proof. See Appendix 7.

We know that the first innovator gains a competitive advantage since $f_{FD}^{t} > f_{FF}^{t} > f_{DF}^{t} = f_{DD}^{t}$. Because of this competition between firms, private diffusion is not very important and the first innovator adopts this cleaner technology later than what is socially desirable. Since $W_{FD}^{t} > W_{DD}^{t}$, the regulator prefers that the first innovator innovates earlier $(t_{1s} < t_{1}^{*})$ in order to produce more than the second

($q^t_{FD} > q^t_{DF}$) and pollutes less ($x^t_{FD} < x^t_{DF}$) in the period $\left[t_{1s},\, t_{2s}\right]$, while reducing the cost of innovation of the second innovator by delaying its date of adoption ($t_{2s} > t^*_2$)[7]. The socially optimal adoption date of the first innovator can be immediate (equal to zero) if condition (3) is an equality (see (A13)). This is the case when the cost of immediate adoption b is not very high. However, the optimal private adoption date of the first innovator cannot be immediate[8] because $0 \le t_{1s} < t^*_1$.

If the regulator prefers to change the private adoption dates of firms (especially of the first innovator), he can do it by giving an innovation subsidy to each firm equal to the loss incurred by that change in the adoption date.

5. Conclusion

We study taxation of pollution as a tool inducing firms to adopt a cleaner production technology. Two symmetric firms are located in a country. Firms by-produce pollution along with the same good sold on the market, and can adopt a cleaner technology within a time t by supporting an actualized cost ρ(t) which decreases exponentially. In our framework, the cost of instantaneous innovation is relatively high and the discount rate is sufficiently low. The incentive to innovate is provided by the environmental policy consisting of taxing pollution.

Firms choose their optimal adoption dates by maximizing their individual intertemporal payoffs, and the regulator chooses the optimal tax parameter by maximizing the intertemporal social welfare. If the tax parameter is well chosen, then firms will adopt the less polluting innovation at finite but different dates. This innovation enables them to produce more, pollute less, pay lower emission tax and it increases the (intertemporal) profit and social welfare. Furthermore, we have

[7] This result is contrary to the one reached by Carraro and Topa (Theorem 3).

[8] In this paper, we usually compare the socially optimal levels to the private ones (derived from the competition between firms and given the optimal tax parameter), but if we want to restrict ourselves uniquely to the latter, we can replace condition (3) by $\frac{f^t_{FD} - f^t_{DD}}{mr} \le b$. When this last inequality is an equality, then the optimal private adoption date of the first innovator is instantaneous (because of (A5)).

established that this technological diffusion is really socially optimal and that the social adoption date of the first innovator is earlier than the private one, and the contrary for the second innovator. Subsidies may be used to move to the socially optimal dates of innovation.

Notice that we have dealt with an infinite horizon thus avoiding too hard computations, but our results remain valid for a sufficiently long finite horizon.

A possible extension of this work is to consider that damages caused to the environment are due to the stock of pollution rather than to the flow of emissions. It is also interesting to know whether technological diffusion is preserved when there are more than two firms or not.

Acknowledgements

I would like to thank Simon Anderson, John Conley, Chokri Dridi, Drew Fudenberg and Myrna Wooders for their useful suggestions. All remaining errors are of my responsibility.

Appendix 1

DD situation without taxation

$$q_1^* = q_2^* = \frac{a-c}{3b} = q_{DD} \quad , \quad Q^* = \frac{2(a-c)}{3b} = Q_{DD} \quad , \quad p^* = p(Q^*) = \frac{a+2c}{3} = p_{DD}$$

$$a_1^* = a_2^* = 0 = a_{DD} \quad , \quad x_1^* = x_2^* = \frac{k(a-c)}{3b} = x_{DD} \quad , \quad X^* = \frac{2k(a-c)}{3b} = X_{DD}$$

$$\Pi_1^* = \Pi_2^* = \frac{(a-c)^2}{9b} = f_{DD} \quad , \quad \Phi_{DD} = 2f_{DD} = \frac{2(a-c)^2}{9b}$$

The emission / output ratio is $(x/q)_{DD} = k$.

DD situation with taxation

We define $b^{'} = b + vk^2$, then :

$$q_1^* = q_2^* = \frac{a-c}{3b^{'}} = q_{DD}^t \quad , \quad Q^* = \frac{2(a-c)}{3b^{'}} = Q_{DD}^t \quad , \quad p^* = p(Q^*) = \frac{a\,(b+3vk^2)+2bc}{3b^{'}} = p_{DD}^t$$

$$a_1^* = a_2^* = 0 = a_{DD}^t \quad , \quad x_1^* = x_2^* = \frac{k(a-c)}{3b^{'}} = x_{DD}^t \quad , \quad X^* = \frac{2k(a-c)}{3b^{'}} = X_{DD}^t$$

$$\Pi_1^* = \Pi_2^* = \frac{(a-c)^2}{9b^{'}} = f_{DD}^t \quad , \quad \Phi_{DD}^t = 2f_{DD}^t = \frac{2(a-c)^2}{9b^{'}}$$

The emission / output ratio is $(x/q)_{DD}^t = k$.

FF situation with taxation

$$q_1^* = q_2^* = \frac{a-c-d}{3b} = q_{FF}^t \quad , \quad Q^* = \frac{2(a-c-d)}{3b} = Q_{FF}^t \quad , \quad p^* = p(Q^*) = \frac{a+2(c+d)}{3b} = p_{FF}^t$$

$$a_1^* = a_2^* = \frac{vk^2(a-c)-b^{'}d}{3bvk} = a_{FF}^t \quad , \quad A^* = \frac{2\left[vk^2(a-c)-b^{'}d\right]}{3bvk} = A_{FF}^t$$

$$x_1^* = x_2^* = \frac{d}{3vk} = x_{FF}^t \quad , \quad X^* = \frac{2d}{3vk} = X_{FF}^t$$

$$\Pi_1^* = \Pi_2^* = \frac{(a-c-d)^2}{9b} + \frac{d^2}{9vk^2} = f_{FF}^t \quad , \quad \Phi_{FF}^t = 2f_{FF}^t = \frac{2(a-c-d)^2}{9b} + \frac{2d^2}{9vk^2}$$

The emission / output ratio is $(x/q)_{FF}^t = \dfrac{bd}{vk(a-c-d)} < k$.

$$a_{FF}^t > 0 \;\; iff \;\; vk^2(a-c) > b^{'}d \quad i.e. \quad v > v_{A1} = \frac{bd}{k^2(a-c-d)} \qquad \text{(A1)}$$

Therefore, the tax parameter must be sufficiently high to induce firms to abate a strictly positive amount of emissions. The minimum tax parameter is a decreasing function of α and k, and is an increasing function of c and d.

FD situation with taxation

$$q_1^* = \frac{a-c}{3b^{'}} = q_{DF}^t \quad , \quad q_2^* = \frac{vk^2(a-c)-b^{'}d}{2bb^{'}} + \frac{a-c}{3b^{'}} = q_{FD}^t \quad , \quad Q^* = \frac{vk^2(a-c)-b^{'}d}{2bb^{'}} + \frac{2(a-c)}{3b^{'}} = Q_{FD}^t$$

$$p^* = p(Q^*) = \frac{(2b+3vk^2)a+(4b+3vk^2)c+3b^{'}d}{6b^{'}} = p_{FD}^t$$

$$a_1^* = 0 = a_{DF}^t \quad , \quad a_2^* = \frac{vk^2(a-c)-b^{'}d}{2bvk} = a_{FD}^t = A_{FD}^t$$

$$x_1^* = \frac{k(a-c)}{3b^{'}} = x_{DF}^t \quad , \quad x_2^* = \frac{3b^{'}d-vk^2(a-c)}{6b^{'}vk} = x_{FD}^t \quad , \quad X^* = \frac{vk^2(a-c)+3b^{'}d}{6b^{'}vk} = X_{FD}^t$$

$$\Pi_1^* = \frac{(a-c)^2}{9b^{'}} = f_{DF}^t \quad , \quad \Pi_2^* = \frac{\left[vk^2(a-c)-b^{'}d\right]^2}{4bb^{'}vk^2} + \frac{(a-c)^2}{9b^{'}} = f_{FD}^t$$

$$\Phi_{FD}^t = f_{DF}^t + f_{FD}^t = \frac{\left[vk^2(a-c)-b^{'}d\right]^2}{4bb^{'}vk^2} + \frac{2(a-c)^2}{9b^{'}}$$

Firm 1's emission / output ratio is $(x / q)_{DF}^{t} = k$.

Firm 2's emission / output ratio is $(x / q)_{FD}^{t} = \dfrac{b\left[3b^{'}d - vk^2(a-c)\right]}{3v^2k^3(a-c) - 3b^{'}dvk + 2bvk(a-c)} < k$.

$$x_{FD}^{t} \geq 0 \quad iff \quad 3b^{'}d \geq vk^2(a-c) \quad i.e. \quad v \leq v_{A2} = \frac{3bd}{k^2(a-c-3d)} \tag{A2}$$

Hence, an upper limit on the tax parameter must be imposed for the first innovator not to abate more than it pollutes.

By combining conditions (A1) and (A2), we get the feasibility region for the tax parameter :

$$\text{v} \in A(v) = \left]v_{A1} , v_{A2}\right] \tag{A3}$$

Condition (A1) implies that the tax rates verify : $t_{DD} > t_{FD} = t_{DF} > t_{FF}$. It is easy to verify that abatement is an increasing function of v, whereas residual emissions, emission / output ratios and firms profits are decreasing functions of v.

Appendix 2

As expressions (9) and (10) are not differentiable in $t_1 = t_2 = t$, we use them to have the optimal adoption dates when $t_1 \neq t_2$ (diffusion). Then, we use expression (11) to get the optimal simultaneous adoption date. Lastly, we compare the intertemporal payoffs of firms generated by diffusion and simultaneous innovation.

• Suppose that firms decide to innovate at different dates and that firm 1 is the first innovator (the case where firm 2 is the first innovator is symmetric).

Firm 1 maximizes $V_1(t_1,t_2) = g_1^1(t_1,t_2)$ defined in (9) with respect to t_1 :

$$\frac{¶V_1(t_1^*,t_2^*)}{¶t_1} = (f_{DD}^{t} - f_{FD}^{t})e^{-rt_1^*} - r'(t_1^*) = 0 \tag{A4}$$

Expression (2) and the resolution of (A4) give :

$$t_1^* = \frac{\ln\left[(f_{FD}^{t} - f_{DD}^{t}) / bmr\right]}{(1-m)r} > 0 \tag{A5}$$

The above inequality is verified because from (A9) and (A12) we have :

$$(W_{FD}^{t} - W_{DD}^{t}) - (f_{FD}^{t} - f_{DD}^{t}) = \frac{1}{8bb^{'}vk^2}\left[vk^2(a-c) - b^{'}d\right]^2$$

$$+\left[\frac{15(2l - v)k^2}{72b^{'2}}(a-c)^2 - \frac{(2l - v)}{12b^{'}v}(a-c)d + \frac{(18v - l)}{144v^2k^2}d^2\right] > 0$$

Indeed, using (A1) and $v \leq 2\lambda$, we prove that the term between the second brackets is strictly positive.

Thus, from (3) :

$$0 < f_{FD}^{t} - f_{DD}^{t} < W_{FD}^{t} - W_{DD}^{t} \leq bmr \tag{A6}$$

Using (A4) and the expression of ρ(t), second order condition becomes :

$$\frac{\partial^2 V_1(t_1^*, t_2^*)}{\partial t_1^2} = -r(f_{DD}^{t} - f_{FD}^{t})e^{-rt_1^*} - r''(t_1^*) = (1-m)bmr^2 e^{-mrt_1^*} < 0$$

Firm 2 maximizes $V_2(t_1, t_2) = g_2^2(t_1, t_2)$, defined in (10) expressed for it, with respect to t_2 :

$$\frac{\partial V_2(t_1^*, t_2^*)}{\partial t_2} = (f_{DF}^{t} - f_{FF}^{t})e^{-rt_2^*} - r'(t_2^*) = 0 \tag{A7}$$

Solving (A7) gives :

$$t_2^* = \frac{\ln\left[(f_{FF}^{t} - f_{DF}^{t}) / bmr\right]}{(1-m)r} \tag{A8}$$

Using (A7) and the expression of ρ(t), second order condition becomes :

$$\frac{\partial^2 V_2(t_1^*, t_2^*)}{\partial t_2^2} = -r(f_{DF}^{t} - f_{FF}^{t})e^{-rt_2^*} - r''(t_2^*) = (1-m)bmr^2 e^{-mrt_2^*} < 0$$

From (A9), we have $\frac{\partial t_1^*}{\partial v} < 0$, $\frac{\partial t_2^*}{\partial v} < 0$ and $0 < t_1^* < t_2^* < +\infty$ as we have supposed.

Results in Appendix 1 give :

$$f_{FD}^{t} - f_{DD}^{t} = \frac{\left[vk^2(a-c) - b'd\right]^2}{4bb'vk^2} > 0 \text{ and } f_{FF}^{t} - f_{DF}^{t} = \frac{\left[vk^2(a-c) - b'd\right]^2}{9bb'vk^2} > 0 \tag{A9}$$

• Suppose that firms decide to innovate simultaneously. Then, each firm i maximizes $V_i(t_1, t_2) = V_i(t) = g(t)$ defined in (11) with respect to τ. We obtain[9]:

$$t^* = \frac{\ln\left[(f_{FF}^{t} - f_{DD}^{t}) / bmr\right]}{(1-m)r} \tag{A10}$$

• In the following we will prove that the case in which firms innovate simultaneously is not a Nash equilibrium of theinnovation game.

First, we remark that $t_2^* = t^*$ (since $f_{DD}^{t} = f_{DF}^{t}$) meaning that the second innovator (firm 2) adopts at the same date of simultaneous innovation. From (10) and (11) expressed for firm 2, we get $V_2(t_1^*, t_2^*) = V_2(t^*)$. Thus, each firm is indifferent between being the second innovator or innovating simultaneously. Concerning the first innovator (firm 1), we should compare $V_1(t_1^*, t^*)$ to $V_1(t^*)$. Using (2), (9) and (11) we get :

$$V_1(t_1^*, t^*) - V_1(t^*) = \left[\frac{1}{r}(f_{FD}^{t} - f_{DD}^{t})e^{-rt_1^*} - be^{-mrt_1^*}\right] + \left[\frac{-1}{r}(f_{FD}^{t} - f_{DD}^{t})e^{-rt^*} + be^{-mrt^*}\right]$$

Using (A5), (A10) and from (A9) : $f_{FD}^{t} - f_{DD}^{t} = \frac{9}{4}(f_{FF}^{t} - f_{DD}^{t})$, we obtain :

[9] By using the first order condition, second order condition is satisfied.

$$V_1(t_1^*, t^*) - V_1(t^*) = b\left(\frac{f_{FF}^t - f_{DD}^t}{bmr}\right)^{\frac{m}{m-1}} \left[(m-1)\left(\frac{9}{4}\right)^{\frac{m}{m-1}} - \frac{9}{4}m + 1\right]$$

The above difference is strictly positive iff f(m)>0, where function f is defined by :

$$f(x) = (x-1)e^{\frac{x}{x-1}\ln\frac{9}{4}} - \frac{9}{4}x + 1 \quad , \quad \forall x > 1$$

with $f'(x) = \left(1 - \frac{\ln(9/4)}{x-1}\right)e^{\frac{x}{x-1}\ln\frac{9}{4}} - \frac{9}{4}$ and $f''(x) = \frac{(\ln(9/4))^2}{(x-1)^3}e^{\frac{x}{x-1}\ln\frac{9}{4}} > 0$

So, f' is strictly increasing with $\lim_{1^+} f'(x) = -\infty$ and $\lim_{+\infty} f'(x) = 0$, then f'(x) < 0 i.e. f is strictly decreasing with $\lim_{1^+} f(x) = +\infty$ and $\lim_{+\infty} f(x) = \frac{9}{4}\left(\ln(9/4) - 1\right) + 1$ >0. Therefore, $f(x) > 0$, $\forall x > 1$.

We can affirm that each firm prefers to be the first innovator than innovating simultaneously. Therefore, the case in which the two firms innovate simultaneously is not a Nash equilibrium because one firm can deviate by being the first adopter. This innovation game is then characterized by two possible Nash equilibria, in which one firm innovates before the other and gains more.

Appendix 3

Using (13), (14) and the results of Appendix 1, we get :

$$CS_{DD} = \frac{2(a-c)^2(b-2lk^2)}{9b^2} \quad , \quad W_{DD} = \frac{4(a-c)^2(b-lk^2)}{9b^2}$$

$$CS_{DD}^t = \frac{2(a-c)^2\left[b + 2k^2(v-l)\right]}{9b'^2} \quad , \quad W_{DD}^t = \frac{2(a-c)^2\left[2b + k^2(3v-2l)\right]}{9b'^2}$$

$$CS_{FF}^t = \frac{2(a-c-d)^2}{9b} + 4\left(\frac{d}{3vk}\right)^2(v-l) \,, \quad W_{FF}^t = \frac{4(a-c-d)^2}{9b} + 2\left(\frac{d}{3vk}\right)^2(3v-2l)$$

$$CS_{FD}^t = CS_{DF}^t = \frac{\left[vk^2(a-c) - b'd\right]^2}{8bb'^2} + \frac{(a-c)\left[vk^2(a-c) - b'd\right]}{3b'^2} + \frac{2b(a-c)^2}{9b'^2} + \frac{(v-l)\left[vk^2(a-c) + 3b'd\right]^2}{36(b'vk)^2}$$

$$W_{FD}^t = W_{DF}^t = CS_{FD}^t + \Phi_{FD}^t$$

Notice that $W_{DD} \geq 0$ iff $l \leq \frac{b}{k^2}$. This suggests the introduction of the environmental tax when λ is too high for the social welfare to be positive. Otherwise, the activity of firms is not desirable.

Appendix 4

The results of Appendix 3 give :

$$W_{FF}^{t} - W_{FD}^{t} = \frac{5}{72bb'vk^2}\left[vk^2(a-c)-b'd\right]^2 + \frac{2l-v}{72}\left[\frac{k^2}{b'^2}(a-c)^2 + \frac{6}{b'v}(a-c)d - \frac{7}{v^2k^2}d^2\right] \quad \text{(A11)}$$

Using condition (A1) and v ≤ 2λ, we prove that the term between the second brackets is strictly positive implying :

$$W_{FF}^{t} - W_{FD}^{t} > 0$$

Similarly :

$$W_{FD}^{t} - W_{DD}^{t} = \frac{27}{72bb'vk^2}\left[vk^2(a-c)-b'd\right]^2 + \left[\frac{15(2l-v)k^2}{72b'^2}(a-c)^2 - \frac{(2l-v)}{12b'v}(a-c)d + \frac{(18v-l)}{144v^2k^2}d^2\right] \text{(A12)}$$

We prove $W_{FD}^{t} - W_{DD}^{t} > 0$.

It is easy to verify that $W_{DD}^{t} \geq 0$ iff $v \geq \frac{2}{3}l - \frac{2b}{3k^2}$.

Appendix 5

We proceed as follows : First, we establish a Lemma which shows that when the regulator decides to tax pollution, the optimal tax parameter maximizing intertemporal social welfare W^t is $\bar{v} = \frac{4}{3}l$. Then, we compare $W^t(\bar{v})$ with W^0 (no taxation), to determine whether the regulator should tax emissions or not. This comparison will show that $W^t(\bar{v}) > W^0$ occurs in the interval $\left]l_1, l_{A2}\right]$. It is then necessary to compare this interval with A(v) (given by (A3), expressed in terms of λ), for the solution to be feasible.

Lemma 1. Suppose (A1),(A2), v£2 l and the discount rate sufficiently close to zero, then the intertemporal social welfare W^t is maximized by $\bar{v} = \frac{4}{3}l$.

Proof : The innovation game between firms results in both firms adopting the cleaner technology at different dates when (A1),(A2) and v≤2λ are verified. Furthermore, suppose that the discount rate is sufficiently close to zero. As the time horizon is infinite, the maximization of W^t is reduced to the maximization of W_{FF}^{t} since :

$$\lim_{r\to 0^+}\int_{t_2}^{+\infty} W_{FF}^{t} e^{-rt}\,dt = +\infty \quad \text{but} \quad \lim_{r\to 0^+}\left[\int_0^{t_1} W_{DD}^{t} e^{-rt}\,dt + \int_{t_1}^{t_2} W_{FD}^{t} e^{-rt}\,dt - r(t_1) - r(t_2)\right] < +\infty$$

Maximizing $W_{FF}^{t} = \frac{4(a-c-d)^2}{9b} + 2\left(\frac{d}{3vk}\right)^2(3v-2l)$ with respect to v, we get $\bar{v} = \frac{4}{3}l$.

Using the previous argument to compare $W^t(\overline{v})$ and W^0, we simply need to compare $W^t_{FF}(\overline{v})$ and W_{DD}.

$$W^t_{FF}(\overline{v}) - W_{DD} = \frac{16k^4(a-c)^2 l^2 - 16bdk^2[2(a-c)-d]l + 9b^2d^2}{36b^2k^2 l}$$

Then $W^t_{FF}(\overline{v}) - W_{DD} > 0$ iff $l \in \left]0, l_0\right[\cup \left]l_1, +\infty\right[$ where :

$$l_0 = \frac{bd\left[4(a-c) - 2d - \sqrt{[7(a-c)-2d](a-c-2d)}\right]}{4k^2(a-c)^2} > 0$$

$$l_1 = \frac{bd\left[4(a-c) - 2d + \sqrt{[7(a-c)-2d](a-c-2d)}\right]}{4k^2(a-c)^2} > l_0$$

The above condition on λ (or v) must be compatible with conditions (A1) and (A2).

If $v = \overline{v} = \frac{4}{3}l$, then conditions (A1) and (A2) become $l \in \left]l_{A1}, l_{A2}\right]$ with :

$$l_{A1} = \frac{3bd}{4k^2(a-c-d)} < l_{A2} = \frac{9bd}{4k^2(a-c-3d)}$$

Using α - c > 3d , we get the following ranking :

$$0 < l_0 < l_{A1} < l_1 < l_{A2}$$

So, $W^t(\overline{v}) > W^0$ iff $l \in \left]l_1, l_{A2}\right]$.

Appendix 6

If (A1), (A2) and $v \le 2\lambda$ are satisfied, this implies that the regulator decides to tax emissions and that firms will innovate at different dates. But what is socially optimal ? The adoption of the new technology by firms at different dates and what are these dates, or the adoption at the same date ? To answer these questions, we will determine the socially optimal adoption dates and compare the intertemporal social welfare in both cases.

• Suppose that firms adopt the new technology at different dates. What are these socially optimal adoption dates ?

The regulator maximizes $W^t(t_1, t_2)$ given by (16) with respect to t_1 and t_2. We get [10] :

$$t_{1s} = \frac{\ln\left[(W^t_{FD} - W^t_{DD})/bmr\right]}{(1-m)r} \geq 0 \;\; , \;\; t_{2s} = \frac{\ln\left[(W^t_{FF} - W^t_{FD})/bmr\right]}{(1-m)r} \qquad \text{(A13)}$$

Inequality (3) implies $t_{1s} \geq 0$. Using (A12), (A11) , and then (A1) and $v \le 2\lambda$ for the following terms without brackets in (A14), we get :

[10] Second order condition is verified.

$$(W_{FD}^t - W_{DD}^t) - (W_{FF}^t - W_{FD}^t) = \frac{11}{36bb'vk^2}\left[vk^2(a-c) - b'd\right]^2 + \frac{7(2l-v)k^2}{36b'^2}(a-c)^2 - \frac{12(2l-v)}{72b'v}(a-c)d + \frac{4v+27l}{144v^2k^2}d^2 > 0 \qquad \text{(A14)}$$

So, $0 \le t_{1s} < t_{2s} < +\infty$.

• Suppose that firms adopt the new technology simultaneously. What is the socially optimal adoption date ?

The regulator maximizes $W^t(t)$ given by (17) with respect to τ. We get [8] :

$$t_s = \frac{\ln\left[(W_{FF}^t - W_{DD}^t)/2bmr\right]}{(1-m)r}$$

But $W_{FF}^t - W_{DD}^t = (W_{FF}^t - W_{FD}^t) + (W_{FD}^t - W_{DD}^t)$, and using inequality (A14), we obtain :

$$W_{FF}^t - W_{FD}^t < \frac{W_{FF}^t - W_{DD}^t}{2} < W_{FD}^t - W_{DD}^t$$

Thus, we have :

$$0 \le t_{1s} < t_s < t_{2s} < +\infty$$

•To know whether diffusion is socially preferred or not, we will compare $W^t(t_{1s}, t_{2s})$ to $W^t(t_s)$.

$$W^t(t_{1s}, t_{2s}) = \int_0^{t_{1s}} W_{DD}^t e^{-rt}\,dt + \int_{t_{1s}}^{t_s} W_{FD}^t e^{-rt}\,dt + \int_{t_s}^{t_{2s}} W_{FD}^t e^{-rt}\,dt + \int_{t_{2s}}^{+\infty} W_{FF}^t e^{-rt}\,dt - r(t_{1s}) - r(t_{2s})$$

$$W^t(t_s) = \int_0^{t_{1s}} W_{DD}^t e^{-rt}\,dt + \int_{t_{1s}}^{t_s} W_{DD}^t e^{-rt}\,dt + \int_{t_s}^{t_{2s}} W_{FF}^t e^{-rt}\,dt + \int_{t_{2s}}^{+\infty} W_{FF}^t e^{-rt}\,dt - 2r(t_s)$$

$$\begin{aligned} W^t(t_{1s}, t_{2s}) - W^t(t_s) &= \left[\frac{1}{r}(W_{FD}^t - W_{DD}^t)e^{-rt_{1s}} - r(t_{1s})\right] + \left[\frac{1}{r}(W_{FF}^t - W_{FD}^t)e^{-rt_{2s}} - r(t_{2s})\right] \\ &\quad - \left[\frac{1}{r}(W_{FF}^t - W_{DD}^t)e^{-rt_s} - 2r(t_s)\right] \\ &= b(m-1)\left[\left(\frac{W_{FF}^t - W_{FD}^t}{bmr}\right)^{\frac{m}{m-1}} + \left(\frac{W_{FD}^t - W_{DD}^t}{bmr}\right)^{\frac{m}{m-1}} - 2\left(\frac{W_{FF}^t - W_{DD}^t}{2bmr}\right)^{\frac{m}{m-1}}\right] \end{aligned}$$

The function $h(x) = x^{\frac{m}{m-1}}, \forall\, x > 0$ is convex since h''(x)>0 i.e. 2h($\frac{x+y}{2}$)<h(x)+h(y) ∀x,y>0. As a consequence $W^t(t_{1s}, t_{2s}) - W^t(t_s) > 0$.

Therefore, the regulator prefers that firms innovate at different dates.

Appendix 7

- Inequality (A6) is equivalent to $0 \le t_{1s} < t_1^* < +\infty$.
- From (A11) and (A9) we have :

$$(W_{FF}^t - W_{FD}^t) - (f_{FF}^t - f_{DF}^t) = \frac{-3b'v - 7(2l - v)b}{72bv^2k^2}d^2 + \frac{6b'v + 6(2l - v)b}{72bb'v}(a-c)d + \frac{-3b'vk^2 + (2l - v)bk^2}{72bb'^2}(a-c)^2 \quad \text{(A15)}$$

The lowest root of the above equation in d is $d_1 = \frac{vk^2}{b'}(a-c)$.

If $d < d_1$, then $(W_{FF}^t - W_{FD}^t) - (f_{FF}^t - f_{DF}^t) < 0$ i.e. $t_{2s} > t_2^*$.

But (A1) is equivalent to $d < d_1$. Thus, $t_{2s} > t_2^*$.

Finally, we reach the following ranking :

$$0 \le t_{1s} < t_1^* < t_2^* < t_{2s} < +\infty$$

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