TOWARD A QUANTITATIVE ANALYSIS OF INDUSTRIAL CLUSTERS II: SHAPLEY VALUE, ENTROPY, AND OTHER FUZZY MEASURES

by

Chokri Dridi and Geoffrey J.D. Hewings

REAL 02-T-6 June, 2002

Toward a Quantitative Analysis of Industrial Clusters II: Shapley Value, Entropy, and Other Fuzzy Measures*

Chokri Dridi
Department of Agricultural and Consumer Economics, and Regional Economics Applications Laboratory, University of Illinois at Urbana-Champaign
cdridi@uiuc.edu

Geoffrey J.D. Hewings
Regional Economics Applications Laboratory, University of Illinois at Urbana-Champaign
hewings@uiuc.edu

Abstract: This paper is a continuation of research in Dridi and Hewings (2002b) that explored the application of fuzzy cluster analysis. In this paper, we introduce a set of tools based on fuzzy set theory to quantitatively examine the structure of input-output based industrial clusters and relations between clusters, and to identify important sectors using a modified Shapley value and entropy measures. The methodology is illustrated with an application using 1990 US input-output data.

JEL Classification: C45, C67, C71, R15
Keywords: Entropy, Industrial clusters, Fuzzy sets, Fuzzy measure, Shapley value.

Correspondence should be addressed to:

Chokri Dridi
Regional Economics Applications Laboratory
University of Illinois
607 S. Mathews #220
Urbana, IL 61801-3671 USA

Tel: (217) 244-7226 Fax: (217) 244-9339

* An earlier draft of this paper was presented at the 42nd Annual Meeting of the Western Regional Science Association-Rio Rico, Arizona (February 26-March 1, 2003), comments from conference participants are acknowledged. The usual disclaimer applies.

TOWARD A QUANTITATIVE ANALYSIS OF INDUSTRIAL CLUSTERS II: SHAPLEY VALUE, ENTROPY, AND OTHER FUZZY MEASURES*

1. INTRODUCTION:

Porter's (1990) work although not devoted to the formal development of cluster analysis highlighted the importance of identifying and analyzing industry clusters for regional development policies. However, missing from the analysis were transparent methods that could be used to identify and analyze industry clusters and this has led to difficulties in the empirical application of an attractive conceptualization (Feser, 1998). One might suggest, as Feser and Luger (2001) noted, that the popularity of cluster analysis is due to the fact that "it is couched in the more accessible verbal language of business competitiveness rather than in the mathematically refined vernacular of urban and regional economics." However, without delving into what could quickly become an ideological debate, we believe that the best qualitative analysis at best touches only the visible surface of complex phenomena. On the other hand, many sophisticated quantitative analyses could be criticized for the absence of connections to issues of policy relevance.

In fact, the lack of a flexible quantitative approach led many analysts, researchers, and consultants to rely more heavily on *expert systems* based on interviews with industry practitioners and decision-makers. Although helpful in interpreting the results, expert systems probably have a more limited role in the definition of clusters. By using fuzzy mathematics, we are hoping to fill the void that exists between uncompromising methods of cluster identification

* An earlier draft of this paper was presented at the 42nd Annual Meeting of the Western Regional Science Association-Rio Rico, Arizona (February 26-March 1, 2003), comments from conference participants are acknowledged. The usual disclaimer applies.

like Ward's (1963) and similar methods and the imprecise and subjective assertions that might result from an exclusive reliance on expert systems.

Bergman and Feser (1999), and Feser and Bergman (2000) identified three objectives that cluster identification has to consider. The competing objectives were: "derive a set of clusters based on the most significant linkages as revealed in the I/O data matrix", identify "a set of mutually exclusive clusters", and "investigate the linkages both between clusters as well as between industries within each cluster". Regarding Bergman and Feser's second objective, Dridi and Hewings (2002a and 2002b) revealed that crisp industrial clusters result in the loss of much important information about the clusters' structures. By disregarding, with good reasons, the second objective it is possible to identify *primary* and *secondary* industries, relations between clusters, and relations between industries within the same cluster. The method we suggest here does not seem to present competing objectives as it relies first on a decomposition of interindustry flows using a decomposition method known as dual scaling (Nishisato, 1980, 1994) and then use the derived row and column weights to identify fuzzy clusters (Zadeh, 1965 and Kaufman and Rousseeuw, 1990) for the sales and purchases profiles.

Using the cardinality, subsethood, involvement, distance measures, and entropy measures, in the next section, we examine the structure and the relation between the clusters obtained from Dridi and Hewings (2002b). In the third section, we reformulate a popular tool from game theory, the Shapley value, to assess the importance of industries from the sales and purchases profiles. In the fourth section, an illustration using 1990 US input-output data helps in understanding how the results of the previous sections provides insights into uncovering the structure of an economy through the industry clusters lens.

2. Clusters' Structure: Quantitative Analysis

In Dridi and Hewings (2002b), a presentation of basic notions about fuzzy set theory applied to industrial clusters analysis was provided and membership values to each fuzzy clusters where found for each industry in the US economy; a brief review is provided here to assist in the exposition.

Based on Zadeh (1965), let *X* be a reference finite and countable space of points, where a given point is denoted by *x*. A fuzzy set (or subset) $A \subseteq X$ is characterized by a real valued membership function $\mu_A(x)$ that associates with each point x a value from a real interval usually normalized to $[0,1]$. The fuzzy subset A of X is a set of ordered pairs $\{(x \mid \mu_A(x)); \forall x \in X\}$, where $\mu_A(x)$ is the grade or degree of membership of *x* in *A*.

$$\mu_A : X \rightarrow [0,1] \tag{1}$$

If we denote by A_k, $\forall k = 1,...,K$, all the subsets of the universal set *X*, then the following properties always hold:

$$\begin{cases} \mu_{A_k}(x) \in [0,1]; \forall x \in X, \forall k = 1,...K \\ \sum_{k=1}^{K} \mu_{A_k}(x) = 1 \end{cases} \tag{2}$$

2.1. Cardinality

The idea behind the cardinality measure is to assess the strength of a set as being a sum of all its memberships.

Let X be the finite and countable universal set of points $x_i, \forall i \in I = \{1,...,N\}$ (here industries), and A_k, $\forall k \in C = \{1,...,K\}$ fuzzy subsets of *X*. We use the notation card(A_k) or $|A_k|$ to refer to the *cardinality* of the fuzzy set A_k, and is defined as (Lootsma, 1997):

$$\text{card}(A_k) = \sum_i \mu_{A_k}(x_i) \qquad ; \ \forall k \in C \tag{3}$$

The properties of this measure are summarized as follows:

a. An empty set has no cardinality: $\text{card}(\varnothing) = 0$.

b. The cardinality of the universal set X is equal to its order or cardinality. Since $\sum_k A_k(x_i) = 1, \forall i = 1,\ldots,n$ then $\text{card}(X) = \sum_k \sum_i \mu_{A_k}(x_i) = n$.

c. The cardinality of the complement of a set A_k is the cardinality of the universal set diminished by the cardinality of A_k.

$$\text{card}\left(A_k^C\right) = \sum_i \left(1 - \mu_{A_k}(x_i)\right) = \text{card}(X) - \text{card}(A_k) \qquad ; \ \forall k \in C \tag{4}$$

The cardinality of the general intersection or general union of sets is a straightforward application of the above definition and classical operation on fuzzy sets.

d. $\text{card}\left(\bigcap_{k \in c} A_k\right) = \sum_i \min\left\{\mu_{A_k}(x_i); \forall k \in C\right\}$.

e. $\text{card}\left(\bigcup_{k \in c} A_k\right) = \sum_i \max\left\{\mu_{A_k}(x_i); \forall k \in C\right\}$.

f. The cardinality of the difference of the fuzzy sets A and B is the cardinality of A diminished by the cardinality of $A \cap B$; $\text{card}(A \setminus B) = \text{card}(A) - \text{card}(A \cap B)$; if A and B are disjoint sets ($A \cap B = \varnothing$) then $\text{card}(A \setminus B) = \text{card}(A)$.

g. The cardinality of the symmetric difference of the fuzzy sets A and B is:

$$\begin{aligned} \text{card}(A \Delta B) &= \text{card}\left((A \setminus B) \cup (B \setminus A)\right) \\ &= \text{card}\left(\max\left(\mu_{A \setminus B}(x_i), \mu_{B \setminus A}(x_i)\right)\right) \end{aligned} \qquad ; \forall i = 1,\ldots,n \tag{5}$$

2.2. Sectors' Involvement in Fuzzy Clusters

One of the dominant features of many approaches to uncovering the structure of economies has been the search for key or analytically important sectors (for a recent review, see Sonis *et al*., 2000). Continuing this search, it turns out that among the information that can be retrieved about the structure of a cluster is the assessment of the relative importance of a sector in the cluster; this can be assessed through what we term *involvement*. The involvement of a an industry x_i in a fuzzy cluster *A* is defined by:

$$\mathrm{Inv}_A\left(x_i\right)=\frac{\mu_A\left(x_i\right)}{\mathrm{card}\left(A\right)} \qquad ;\forall i=1,\ldots,n \tag{6}$$

A slicing of industries in a cluster based on their involvement can be done with the following operations:

a. The *Lea*d of a fuzzy set *A* is defined as:

$$\mathrm{Lead}\left(A\right)=\left\{x_i \mid \max\left\{\mathrm{Inv}_A\left(x_i\right);\forall x_i \in X\right\}\right\} \tag{7}$$

b. The *α-Lead* of a fuzzy set *A* is defined as:

$$\alpha\text{-Lead}\left(A\right)=\left\{x_i \mid \mathrm{Inv}_A\left(x_i\right)\geq\alpha;\forall\alpha\in\left[0,1\right],x_i\in X\right\} \tag{8}$$

The *Lead* and *α-Lead* can be useful tools to single out primary and secondary industries that are leading the cluster, or industries around which other industries flock.

c. The *Trail* of a fuzzy set *A* is defined as:

$$\mathrm{Trail}\left(A\right)=\left\{x_i \mid \min\left\{\mathrm{Inv}_A\left(x_i\right);\forall x_i \in X\right\}\right\} \tag{9}$$

d. The *β-Trail* of a fuzzy set *A* is defined as:

$$\beta\text{-Trail}\left(A\right)=\left\{x_i \mid \mathrm{Inv}_A\left(x_i\right)\leq\beta;\forall\beta\in\left[0,1\right],x_i\in X\right\} \tag{10}$$

The *Trail* and *β-Trail* measures serve to identify lagging industries in a cluster depending on the level of the *β-Trail* set, any industries that are neither in the *α-Lead* level set nor the *β-Trail* level set are considered supporting industries.

The use of the involvement indicator and the slicing provided above allows for the identification of primary and secondary industries, then based on the elements of *α-Lead* set of level α one can identify certain clusters as being manufacturing, textile, etc… clusters. What remains to be explored is the degree to which this involvement indicator is sensitive to changes in the structure of the economy and, concomitantly, the way in which changes in the character of the *lead* industry generates changes in the economy revealed through changes in the number and/or composition of clusters.

2.3. Subsethood and Distance between Fuzzy Sets

The subsethood (Lootsma, 1997) is an indicator of the degree that a fuzzy set $B \neq \varnothing$ is a subset of A. The subsethood is similar to the conditional probability and is defined by:

$$\begin{aligned} S\left(B,A\right) &= \frac{\mathrm{card}(B \cap A)}{\mathrm{card}(B)} \\ &= \frac{\sum_i \min\left(\mu_A(x_i), \mu_B(x_i)\right)}{\sum_i \mu_B(x_i)} \end{aligned} \tag{11}$$

Form (11) one can check that if $A \cap B = \varnothing$, then $S(B,A) = 0$ and if $B \subseteq A$ then $S(B,A) = 1$, and in all intermediate cases we have $0 < S(B,A) < 1$. In fact, the subsethood can be viewed as an assessment of cluster B's relative dependence on cluster A.

Another measure of interest is the distance between fuzzy clusters, similarly to the Euclidian distance it is defined as:

$$d\left(A,B\right)=\sqrt{\sum_{i=1}^{n}\left(\mu_A\left(x_i\right)-\mu_B\left(x_i\right)\right)^2} \tag{12}$$

In extreme cases where an industry belongs to one and only one cluster, the maximum distance is $\sqrt{n}$ and its minimum value is 0, this implies that we can normalize (12) by $\sqrt{n}$, which rescales the values for (13) to $\left[0,1\right]$.

$$d'\left(A,B\right)=\sqrt{\frac{\sum_{i=1}^{n}\left(\mu_A\left(x_i\right)-\mu_B\left(x_i\right)\right)^2}{n}} \tag{13}$$

The distance measure provides additional information about the difference in fuzzy clusters when the subsethood measure fails to provide information about the relation between two fuzzy clusters, for instance when $A\cap B=\varnothing$ we have $S(B,A)=0$ no information about the difference (i.e dissimilarity) between clusters is available. In addition, regardless of the subsethood measure, the distance between clusters provides insights into the complementarities or competition between clusters in a spatial setting, where more than one cluster involving the same industries might exist in different locations.

2.4. Fuzzy Clusters, Industries' membership and Entropy Measures

Further comparison of clusters and industries is possible using information theory tools such as Shannon's entropy measure, which gives an assessment of the cluster's informational content (Reza, 1961). Shannon's entropy provides a measure of the average uncertainty, which when translated into a fuzzy sets context gives a measure of average possibility. For an industry i with a membership $\mu_k(x_i)$ over all fuzzy clusters $k=1,...,K$ the entropy or the industries' average membership possibilities measure is given by:

$$I\left(x_i\right)=-\sum_{k=1}^{K}\mu_k(x_i)\log\left(\mu_k(x_i)\right) \tag{14}$$

The higher the value of the entropy measure in (14), more the membership possibilities are numerous, stated differently higher entropy values imply more possibilities (or diversity) in the distribution of the membership values.

Similarly, for the involvement measure defined in (6) allows to compute the entropy of a cluster k as:

$$H\left(k\right)=-\sum_{i=1}^{n}\mathrm{Inv}_k(x_i)\log\left(\mathrm{Inv}_k(x_i)\right) \tag{15}$$

For (15), higher values of entropy portray a higher diversity in cluster structure.

Both entropy measures in (14) and (15) allow for a spatial as well as a temporal comparison of clusters' structure and/or industries' involvement in various clusters.

3. Modified Shapley Value for Fuzzy Clusters:

The Shapley value is a solution concept for an n-person cooperative game introduced by Shapley (1953) as a method to decide on the share of each player in a cooperative game with transferable utility. The use of the Shapley value extends to areas other than game theory and industrial organization. To the best of our knowledge, this paper represents the first attempt to use the Shapley value concept to establish a ranking of industries in an economy when viewed as a collection of more or less dense fuzzy clusters. This kind of characterization is made possible through the adoption of fuzzy set theory to identify clusters (see Dridi and Hewings, 2002b).

The concept of value introduced by Shapley can be used to measure the effect of the exclusion of an industry when applied to input-output based industry fuzzy-clusters, if we consider that the players in a game are the industries and that a coalition is an industry cluster. In this context, the value of each industry is not assessed on the basis of its backward and forward linkages or the volume of inputs and outputs, as in Cella (1984 and 1986), Clements (1990) or

Guilhoto *et al* (1994), but rather based on its economy wide importance when the economy is viewed as a set of fuzzy clusters. This perspective opens up some new possibilities in the interpretation of the structure of an economy. Consider for example, the concept of a field of influence, introduced by Sonis and Hewings (1992); here, a methodology was presented to identify analytically important coefficients based on the impact that a small change in one of them would have system-wide. While the methodology can be extended to consider changes in rows or columns, another possibility would be to view changes in the context of clusters. In this regard, economy-wide importance might be revealed at two-levels – the sub-field of influence, contained within the cluster or set of cluster with which the sector is associated, and the full field of influence that relates changes in one cluster to all clusters. The degree to which the influence is or is not concentrated within the cluster provides a further insight into the *degree of containment* associated with changes in one sector; consider two sector that have the same aggregate, system-wide impact associated with any change in their structure. For one sector, the majority of the impact might be contained within the cluster while for another, the influence within the cluster may only be marginally more important than the influence outside. Of course, the degree of fuzziness in the cluster identification process will provide some initial expectations about the degree of containment - the more[1] fuzzy the cluster, the greater the expectation of a smaller degree of containment. Extending the ideas to a multiregional context, the degree of containment would then have both spatial and sectoral components (see Seo and Hewings, 2002 for an analysis of the interregional spillovers of international trade in a multiregional context that would be a suitable candidate for a new interpretation using this methodology).

[1] To assess the degree of fuzziness of a cluster one could consider the distance between the cluster and its complement, the smaller the distance between the two the fuzzier the cluster (Dumitrescu et al., 2000, p. 185-186).

Let us describe the situation we are analyzing in the context of an economy with a set of industries $I = \{1,...,N\}$ with memberships $\mu_k(x_i); \forall i \in I, k \in C$ to a set of fuzzy clusters $C = \{1,...,K\}$. Let $v_k(S)$ be the value of the subset of industries $S \subseteq I$ belonging to a fuzzy cluster $k \in C$. The marginal value of an industry *i* when removed from a subset of industries *S* in a given cluster *k* is:

$$v_k(S) - v_k(S - \{i\}) \tag{16}$$

At this point, we overlook the fuzzy clusters issue and treat the industries as if they were a coalition of players in a cooperative game with transferable utility and establish Shapley (1953) results first and then introduce the existence of fuzzy clusters to reformulate the Shapley value.

If we consider all possible combinations of industries $S \subseteq I$, then a given subset of industries S with cardinality $|S| = s$ containing industry i exists with a probability of $\gamma_{s,N} = \frac{1}{C_{N-1}^{s-1}} = \frac{(N-s)!(s-1)!}{(N-1)!}$. An industry *i*, for which we seek to compute the *value* or *worth*, is chosen with the probability of $\frac{1}{N}$; therefore Shapley's version for the value of industry *i* is given by (Shubik, 1982):

$$V(i) = \frac{1}{N}\sum_{s=1}^{N}\gamma_{s,N}\sum_{\substack{S\subseteq I \\ |S|=s}}\left(v_k(S) - v_k(S - \{i\})\right) \tag{17}$$

In Yager (2000, 2001), where *fuzzy Shapley entropy* was derived as a fuzzy characterization of uncertainty of the fuzzy measure, the value function was replaced by the membership function. However, the different combinations of clusters from which an industry *i* can be excluded was not considered. Considering that each industry belongs to more than one cluster, i.e. fuzzy clusters then the Shapley value has to be modified to accommodate this fuzziness.

In a given coalition of industries, the industry i can be removed from a variety of subsets of clusters Q of size q with the restriction that each industry must belong to at least one cluster, i.e. the industry i can be removed from up to $k-1$ clusters simultaneously. Each subset of clusters Q of size q exists with a probability of: $\lambda_{q,K} = \frac{1}{C_K^q} = \frac{q!(K-q)!}{K!}$, which transforms (17) into the following fuzzy Shapely value:

$$V(i) = \frac{1}{N}\sum_{q=1}^{K-1}\lambda_{q,K}\sum_{\substack{Q\subseteq C\\|Q|=q}}\sum_{s=1}^{N}\gamma_{s,N}\sum_{\substack{S\subseteq I\\|S|=s}}\left(v_Q(S)-v_Q(S-\{i\})\right) \tag{18}$$

If we define the value function as being the sum of the memberships over all subsets of clusters and coalitions, $v_Q(S) = \sum_{p=1}^{q=|Q|}\sum_{i\in S}\mu_p(x_i)$, and reexamine expression (18), then based on the linearity of $v_Q(.)$ one can conclude that $v_Q(S)-v_Q(S-\{i\}) = \sum_{p=1}^{q=|Q|}\mu_p(x_i)$, which transforms (18) into:

$$V(i) = \frac{1}{N}\sum_{q=1}^{K-1}\lambda_{q,K}\sum_{\substack{Q\subseteq C\\|Q|=q}}\sum_{s=1}^{N}\gamma_{s,N}C_N^s\sum_{p=1}^{q=|Q|}\mu_p(x_i) \tag{19}$$

After simplification of $\frac{C_N^s}{N}\gamma_{s,N} = \frac{1}{s}$, expression (19) becomes:

$$V(i) = \sum_{q=1}^{K-1}\lambda_{q,K}\sum_{\substack{Q\subseteq C\\|Q|=q}}\sum_{s=1}^{N}\frac{1}{s}\sum_{p=1}^{q=|Q|}\mu_p(x_i) \tag{20}$$

In its original formulation (17), the sum of all Shapley index is equal to one and each player's (i.e. industry's) value is a fraction from the continuum $[0,1]$. However, in our context we have

$\sum_{i \in I} V(i) < 1$ because we left out membership values of industry *i* in at least a cluster each time we removed it from a coalition and a subset of clusters. Recall that we removed each industry from up to *k*-1 fuzzy clusters; the total of the Shapley value is not very far from one the difference as will be seen in the next section is less than 5% for the application at hand, and we expect that as the number of industries the difference will tend to zero. The difference $1 - \sum_{i \in I} V(i)$ can be interpreted as being the average value of various combinations of industries that are removed from all but one fuzzy cluster.

4. Insights from the US Input-output Data

To illustrate the results of the previous sections we use the same 1990 US input-output data as in the Dridi and Hewings (2002b).

In tables A.1 and A.2, we provide the various industries' involvement measure and clusters' cardinality. The cardinality of each cluster shows that for the sales profile, cluster R1 is the most important, for the purchases profile cluster C1 is the most important. Depending on the purpose of the study, clusters in the sales and purchases profile can be ranked based on the principle that the cardinality tells us about the importance of each cluster.

The results of the involvement measure in table A.2 show the *leading* industries in various clusters, leading industries display a high level of involvement allowing labeling some clusters based on the industry leading it. For example, if we take cluster R11, then the *Other Transport* industry (ind. 20) seems to be dominating that cluster while showing little involvement in the rest of the clusters, indeed ind. 20 is the *trail* in all other clusters. While the leading industries help in characterizing the cluster they lead, the trailing industries play a supporting role to the rest of the industries in the cluster. A look at table A.1, shows that some sectors do not appear as leading sectors in any cluster, if their activities are similar in nature to the leading industrie(s) then they are contributing to the characterization of the cluster by being secondary industries in the cluster otherwise they are supporting industries. A similar characterization can be done for the purchases profile, in table A.2, *Office & computing machinery* (ind. 16) is the leading in cluster C10, the use of the *α-Lead* allows for a further identification of clusters. Cluster R1, R4, R6, R8, R14, and R15 for the sales profile (table A.1) and clusters C1, C2, and C4 in the purchases profile (table A.2), seem to be made of various industries without any industry or group of similar industries belong to an *α-Lead* set of high level, say 0.4. Cluster with weak industries' involvement would have been totally disregarded in the context of crisp clusters. While it may be difficult to sharply define some clusters, for instance when only few industries have high involvement measure, sometimes what makes the cluster are the diverse supporting industries, who do not necessarily produce or purchase high value goods but rather have frequent transactions involving moderate value goods. Since the purpose of this paper is rather methodological a detailed analysis of the US input-output based cluster structure and relations will probably take twice this space. However, a more detailed study with an even more disaggregated or regional input-output data will reveal a wealth of information about the relations between the clusters that would help in testing many of the perceived relations between clusters to complement some of the more qualitative appraisals offered by Porter (1990, 1998) and others.

A quick assessment of the results provided in tables A.3 and A.4 regarding the dependence of clusters on each other, measured according to (11), shows that in table A.3, except[2] for cluster R5, R9, R11, and R13 all the sales profiles clusters depend one on the other. No cluster is largely dependent on cluster R12. Indeed, in table A.1 the cardinality of those

[2] Safe for the self-dependence.

clusters is the lowest among all the clusters while the interdependent cluster have significantly higher cardinality as if the strength of some clusters comes from the strength of other clusters. In table A.4, only cluster C10 is not dependent and not depending on any other cluster, but clusters C6, C12 while depending on other clusters, they have not dependent clusters. Once more, the cardinality of the clusters confirms the reliance of some clusters on other clusters (see table A.3 and A.4).

The normalized distance values from (13) are given in tables A.5 and A.6, where for the sales and purchases profile most clusters seem to offer little discrepancy with each other. This is due in part to the fact that all the clusters, the sales and purchases profiles alike, are lead at most by one industry with a high involvement in the cluster while the rest of the industries are lagging behind in terms of involvement in the cluster.

Table A.7 shows the entropy of the industries and the clusters in both sales and purchases profiles. The industries' entropy results show that for both sales and purchases profiles, about half the industries have a high entropy while the other half have relatively low entropy. This indicates that the industries with low entropy are not well represented in all clusters, i.e. not diverse enough. For industries with high entropy, their membership function shows more possibility, since all their memberships are rather of the same magnitude. While some diversity in the industries' entropy exists, the clusters' entropy shows little variability, this observation is confirmed by the inter-cluster distance (see tables A.5 and A.6).

In table A.8, we provide the fuzzy Shapely value computed according to (20). The values are in decreasing order for both sales and purchases profiles. In the sales profile, the industries with a value higher then 3 % are in a decreasing order 20, 16, 24, 8, 22, 4, 12, 18, 2, 10, 6, 28, 26, and 14. For the purchases profile, the industries with 3% or higher value are 20, 16, 8, 24, 10, 14, 12, 4, 22, 18, 6, 28, 2, and 26 and they are the same industries as in the sales profile ranking but in a different order. Although for large input-output tables the Shapley value may be computationally greedy, it offers a reliable tool for comparative studies about the importance of some or all the industries in time and space.

<< insert figure 1 here >>

Figure 1, displays the shapely value for each industry in the sales and purchases profiles. A first comment is that for all except the last industry, the shapely value is at least as high for the purchases profile as it is for the sales profile. In addition, the sectors that were globally important using the union operator (the even numbered ones) are found to be more important than the others here as well. Here also, the regularity ceases to exist for the industries after industry 29.

6. Conclusion

The results in this paper and the earlier one (Dridi and Hewings, 2002b) provide confirmation of the need to avoid the problems associated with the inappropriateness of using crisp clusters analysis. Compared to the results obtained using fuzzy clusters and their properties, it is clear that many additional insights can be revealed into the structure of an economy. In particular, the approach offers some new summary results that provide a distinction between the *local* and *global* importance of industries. In the current paper, we pushed further the use of fuzzy set theory to offer ways to quantitatively examine the structure of clusters in an economy through the involvement measure and the slicing of industries based on their relative involvement in fuzzy clusters. In addition, we reformulated the Shapley value, a popular tool in cooperative game theory, to accommodate fuzzy membership values and use it as a reliable indicator of the importance of industries in the economy when viewed as a collection of fuzzy clusters.

In Feser and Luger (2001), the authors argued about the attention cluster analysis is receiving; although the field of inquiry is not new, there is still enormous potential to enhance its popularity in large part because it remains "a highly useful and flexible mode of inquiry." The objective of this research has been to exploit the flexibility that fuzzy logic offers to describe and analyze clusters without loosing sight of two limitations: first the imprecision in the data and also the systematic violation of the 'excluded middle' principle that characterizes fussy sets in society and industrial organization.

There are many potential extensions for this approach; at present, the spatial dimension has not been explored but this can be addressed by applying the methodology to interregional systems. Indeed, the spatial dimension in general and of cluster structure and formation in particular is becoming more and more important in regional science; the approach may offer some resolution to some of the issues raised in Parr *et al.* (2002) where the problems of definition and interpretation of agglomerative effects were discussed. Spatial industrial clustering needs to be examined in a fashion similar to Feser and Sweeney (2000) and Feser *et al.* (2001) where a spatial industry cluster analysis based on input-output data was used in conjunction with spatial information to understand the cluster composition at the regional and sub-regional levels. Now the opportunity exists to develop fuzzy spatial clusters in which the degree of fuzziness may vary across space and sectors.

APPENDIX

Table A.1: Sales profile industries' involvement in fuzzy clusters and cluster cardinality

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15
Ind 1	0.0485	0.0339	0.0307	0.0314	0.0249	0.0317	0.0288	0.0336	0.0220	0.0279	0.0188	0.0246	0.0244	0.0314	0.0304
Ind 2	0.0028	0.4128	0.0019	0.0020	0.0016	0.0019	0.0018	0.0020	0.0014	0.0018	0.0012	0.0016	0.0016	0.0021	0.0020
Ind 3	0.0469	0.0330	0.0306	0.0308	0.0266	0.0314	0.0303	0.0322	0.0223	0.0282	0.0203	0.0247	0.0254	0.0317	0.0302
Ind 4	0.0018	0.0013	0.4425	0.0013	0.0011	0.0014	0.0015	0.0015	0.0011	0.0014	0.0009	0.0012	0.0012	0.0013	0.0013
Ind 5	0.0395	0.0292	0.0293	0.0311	0.0277	0.0319	0.0306	0.0373	0.0253	0.0302	0.0228	0.0281	0.0268	0.0289	0.0290
Ind 6	0.0040	0.0026	0.0022	0.3899	0.0021	0.0027	0.0023	0.0027	0.0018	0.0027	0.0015	0.0021	0.0020	0.0027	0.0028
Ind 7	0.0482	0.0308	0.0258	0.0336	0.0263	0.0350	0.0282	0.0344	0.0226	0.0290	0.0189	0.0260	0.0245	0.0296	0.0303
Ind 8	0.0007	0.0006	0.0006	0.0006	0.5061	0.0006	0.0006	0.0006	0.0005	0.0006	0.0005	0.0006	0.0006	0.0007	0.0006
Ind 9	0.0559	0.0341	0.0267	0.0311	0.0239	0.0292	0.0266	0.0319	0.0204	0.0283	0.0189	0.0234	0.0232	0.0349	0.0320
Ind 10	0.0037	0.0024	0.0023	0.0026	0.0020	0.3917	0.0025	0.0028	0.0018	0.0025	0.0015	0.0022	0.0018	0.0025	0.0026
Ind 11	0.0450	0.0298	0.0291	0.0311	0.0248	0.0330	0.0283	0.0394	0.0237	0.0295	0.0206	0.0266	0.0254	0.0282	0.0299
Ind 12	0.0022	0.0015	0.0018	0.0015	0.0014	0.0017	0.4281	0.0018	0.0013	0.0017	0.0012	0.0014	0.0014	0.0016	0.0016
Ind 13	0.0375	0.0289	0.0335	0.0279	0.0264	0.0296	0.0402	0.0317	0.0259	0.0311	0.0218	0.0271	0.0274	0.0301	0.0295
Ind 14	0.0065	0.0036	0.0037	0.0039	0.0029	0.0042	0.0040	0.3523	0.0026	0.0039	0.0022	0.0031	0.0030	0.0038	0.0038
Ind 15	0.0496	0.0283	0.0306	0.0298	0.0230	0.0310	0.0312	0.0380	0.0224	0.0310	0.0174	0.0259	0.0240	0.0303	0.0297
Ind 16	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0004	0.0003	0.5487	0.0004	0.0003	0.0004	0.0003	0.0003	0.0003
Ind 17	0.0462	0.0282	0.0308	0.0296	0.0241	0.0332	0.0303	0.0358	0.0238	0.0314	0.0183	0.0272	0.0245	0.0306	0.0298
Ind 18	0.0027	0.0017	0.0018	0.0020	0.0016	0.0019	0.0020	0.0021	0.0016	0.4156	0.0013	0.0019	0.0016	0.0019	0.0020
Ind 19	0.0284	0.0293	0.0307	0.0286	0.0334	0.0284	0.0302	0.0268	0.0344	0.0298	0.0385	0.0324	0.0340	0.0279	0.0273
Ind 20	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001	0.0002	0.0001	0.0002	0.0002	0.6095	0.0002	0.0002	0.0001	0.0001
Ind 21	0.0378	0.0284	0.0322	0.0296	0.0271	0.0323	0.0309	0.0304	0.0278	0.0334	0.0223	0.0341	0.0263	0.0283	0.0290
Ind 22	0.0009	0.0007	0.0007	0.0007	0.0007	0.0008	0.0007	0.0007	0.0007	0.0008	0.0006	0.4914	0.0007	0.0007	0.0007
Ind 23	0.0444	0.0284	0.0279	0.0323	0.0262	0.0325	0.0291	0.0308	0.0242	0.0320	0.0193	0.0285	0.0262	0.0316	0.0321
Ind 24	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004	0.0005	0.5140	0.0005	0.0005
Ind 25	0.0561	0.0313	0.0269	0.0314	0.0237	0.0304	0.0281	0.0312	0.0204	0.0287	0.0180	0.0233	0.0229	0.0357	0.0318
Ind 26	0.0057	0.0034	0.0030	0.0035	0.0031	0.0033	0.0030	0.0034	0.0023	0.0033	0.0020	0.0026	0.0026	0.3667	0.0038
Ind 27	0.0715	0.0278	0.0254	0.0315	0.0204	0.0299	0.0268	0.0382	0.0175	0.0279	0.0146	0.0211	0.0200	0.0316	0.0295
Ind 28	0.0041	0.0026	0.0023	0.0029	0.0022	0.0027	0.0024	0.0027	0.0019	0.0027	0.0016	0.0022	0.0021	0.0031	0.3886
Ind 29	0.0622	0.0292	0.0255	0.0308	0.0226	0.0305	0.0263	0.0326	0.0190	0.0285	0.0180	0.0222	0.0216	0.0356	0.0332
Ind 30	0.0577	0.0292	0.0258	0.0333	0.0241	0.0300	0.0265	0.0300	0.0207	0.0295	0.0172	0.0238	0.0229	0.0363	0.0327
Ind 31	0.0451	0.0312	0.0272	0.0325	0.0260	0.0299	0.0277	0.0306	0.0253	0.0306	0.0205	0.0277	0.0260	0.0322	0.0331
Ind 32	0.0763	0.0270	0.0229	0.0308	0.0209	0.0276	0.0239	0.0308	0.0172	0.0269	0.0138	0.0202	0.0196	0.0393	0.0351
Ind 33	0.0671	0.0279	0.0245	0.0309	0.0220	0.0286	0.0259	0.0306	0.0186	0.0281	0.0154	0.0218	0.0219	0.0377	0.0348
Card	*2.7241*	*2.2825*	*2.1671*	*2.3676*	*1.9391*	*2.3618*	*2.2210*	*2.5121*	*1.8029*	*2.2665*	*1.6331*	*1.9888*	*1.9131*	*2.4516*	*2.3687*

Table A.2: Purchases profile industries' involvement in fuzzy clusters and cluster cardinality

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
Ind 1	0.0355	0.0330	0.0321	0.0352	0.0307	0.0266	0.0290	0.0296	0.0290	0.0254	0.0292	0.0254	0.0292	0.0284	0.0326
Ind 2	0.0031	0.3954	0.0024	0.0028	0.0024	0.0020	0.0022	0.0024	0.0023	0.0020	0.0024	0.0020	0.0024	0.0022	0.0026
Ind 3	0.0378	0.0325	0.0319	0.0361	0.0311	0.0254	0.0273	0.0297	0.0280	0.0234	0.0287	0.0245	0.0280	0.0267	0.0383
Ind 4	0.0017	0.0017	0.4323	0.0018	0.0015	0.0014	0.0014	0.0017	0.0017	0.0014	0.0016	0.0015	0.0016	0.0015	0.0017
Ind 5	0.0310	0.0291	0.0302	0.0367	0.0308	0.0285	0.0301	0.0309	0.0313	0.0271	0.0304	0.0298	0.0299	0.0291	0.0284
Ind 6	0.0021	0.0018	0.0016	0.0020	0.4235	0.0016	0.0018	0.0017	0.0016	0.0015	0.0019	0.0015	0.0018	0.0017	0.0018
Ind 7	0.0348	0.0312	0.0268	0.0354	0.0330	0.0285	0.0368	0.0290	0.0268	0.0257	0.0294	0.0257	0.0298	0.0287	0.0303
Ind 8	0.0008	0.0008	0.0007	0.0008	0.0008	0.4899	0.0008	0.0008	0.0008	0.0007	0.0008	0.0007	0.0008	0.0007	0.0008
Ind 9	0.0337	0.0352	0.0310	0.0330	0.0318	0.0281	0.0287	0.0305	0.0287	0.0257	0.0289	0.0257	0.0288	0.0285	0.0333
Ind 10	0.0011	0.0011	0.0010	0.0012	0.0012	0.0011	0.4621	0.0010	0.0010	0.0010	0.0011	0.0010	0.0011	0.0010	0.0011
Ind 11	0.0328	0.0318	0.0296	0.0356	0.0319	0.0274	0.0312	0.0279	0.0284	0.0270	0.0300	0.0291	0.0300	0.0284	0.0311
Ind 12	0.0016	0.0016	0.0016	0.0016	0.0014	0.0013	0.0013	0.4391	0.0018	0.0013	0.0015	0.0016	0.0015	0.0015	0.0016
Ind 13	0.0294	0.0310	0.0331	0.0300	0.0281	0.0273	0.0276	0.0345	0.0381	0.0280	0.0305	0.0268	0.0293	0.0311	0.0292
Ind 14	0.0014	0.0014	0.0015	0.0014	0.0013	0.0012	0.0012	0.0017	0.4461	0.0012	0.0015	0.0015	0.0014	0.0014	0.0014
Ind 15	0.0335	0.0318	0.0327	0.0373	0.0286	0.0247	0.0266	0.0312	0.0320	0.0261	0.0315	0.0251	0.0308	0.0280	0.0312
Ind 16	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006	0.0007	0.0007	0.0007	0.5011	0.0008	0.0007	0.0008	0.0007	0.0007
Ind 17	0.0325	0.0304	0.0316	0.0348	0.0291	0.0259	0.0293	0.0321	0.0316	0.0271	0.0326	0.0259	0.0311	0.0284	0.0299
Ind 18	0.0020	0.0018	0.0017	0.0020	0.0019	0.0015	0.0016	0.0018	0.0018	0.0017	0.4259	0.0016	0.0020	0.0016	0.0017
Ind 19	0.0285	0.0320	0.0312	0.0316	0.0301	0.0301	0.0292	0.0273	0.0320	0.0303	0.0301	0.0293	0.0310	0.0313	0.0304
Ind 20	0.0007	0.0007	0.0007	0.0008	0.0007	0.0006	0.0007	0.0008	0.0008	0.0007	0.0007	0.4993	0.0007	0.0007	0.0007
Ind 21	0.0316	0.0294	0.0331	0.0320	0.0294	0.0262	0.0287	0.0296	0.0289	0.0297	0.0331	0.0283	0.0370	0.0276	0.0288
Ind 22	0.0019	0.0017	0.0016	0.0019	0.0017	0.0015	0.0016	0.0016	0.0016	0.0017	0.0019	0.0015	0.4294	0.0015	0.0016
Ind 23	0.0345	0.0310	0.0293	0.0340	0.0314	0.0268	0.0296	0.0299	0.0289	0.0271	0.0324	0.0260	0.0319	0.0290	0.0302
Ind 24	0.0011	0.0010	0.0010	0.0011	0.0011	0.0009	0.0010	0.0011	0.0011	0.0009	0.0011	0.0009	0.0010	0.4684	0.0010
Ind 25	0.0389	0.0348	0.0298	0.0327	0.0307	0.0266	0.0272	0.0306	0.0279	0.0251	0.0293	0.0272	0.0287	0.0271	0.0339
Ind 26	0.0045	0.0039	0.0035	0.3610	0.0037	0.0029	0.0033	0.0034	0.0033	0.0029	0.0037	0.0031	0.0036	0.0032	0.0036
Ind 27	0.0589	0.0344	0.0266	0.0348	0.0304	0.0237	0.0243	0.0267	0.0258	0.0227	0.0291	0.0225	0.0284	0.0250	0.0316
Ind 28	0.0027	0.0024	0.0022	0.0024	0.0022	0.0020	0.0019	0.0022	0.0021	0.0018	0.0021	0.0019	0.0021	0.0020	0.4066
Ind 29	0.0407	0.0320	0.0288	0.0342	0.0298	0.0268	0.0269	0.0291	0.0277	0.0251	0.0306	0.0296	0.0292	0.0272	0.0327
Ind 30	0.0345	0.0327	0.0293	0.0313	0.0309	0.0292	0.0283	0.0303	0.0288	0.0280	0.0312	0.0279	0.0306	0.0284	0.0311
Ind 31	0.0354	0.0325	0.0282	0.0329	0.0321	0.0278	0.0285	0.0288	0.0280	0.0286	0.0310	0.0261	0.0317	0.0285	0.0318
Ind 32	0.0602	0.0324	0.0265	0.0335	0.0301	0.0267	0.0241	0.0268	0.0259	0.0232	0.0288	0.0219	0.0285	0.0252	0.0313
Ind 33	0.3105	0.0070	0.0056	0.0075	0.0064	0.0050	0.0051	0.0057	0.0054	0.0047	0.0062	0.0044	0.0061	0.0053	0.0069
Card	*2.6480*	*2.3423*	*2.2003*	*2.4724*	*2.2334*	*1.9921*	*2.0909*	*2.1707*	*2.1460*	*1.9533*	*2.2204*	*1.9584*	*2.2089*	*2.0669*	*2.2961*

Table A.3: Sales profile industries' subsethood measures with high levels above 0.5 highlighted

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15
R1	1.0000	0.4949	0.4453	0.5342	0.3523	0.5311	0.4685	0.6038	0.2991	0.4889	0.2343	0.3722	0.3419	0.5757	0.5358
R2	0.5906	1.0000	0.5236	0.5852	0.4202	0.5835	0.5413	0.5894	0.3568	0.5644	0.2795	0.4425	0.4079	0.5893	0.5834
R3	0.5598	0.5515	1.0000	0.5558	0.4425	0.5578	0.5574	0.5592	0.3758	0.5553	0.2944	0.4675	0.4296	0.5588	0.5563
R4	0.6147	0.5642	0.5087	1.0000	0.4052	0.5952	0.5280	0.6115	0.3440	0.5564	0.2695	0.4280	0.3932	0.6066	0.5997
R5	0.4949	0.4946	0.4946	0.4947	1.0000	0.4947	0.4946	0.4947	0.4199	0.4946	0.3290	0.4925	0.4782	0.4948	0.4946
R6	0.6126	0.5639	0.5118	0.5967	0.4061	1.0000	0.5312	0.6112	0.3448	0.5623	0.2702	0.4290	0.3942	0.5963	0.5879
R7	0.5746	0.5563	0.5439	0.5628	0.4318	0.5649	1.0000	0.5696	0.3667	0.5631	0.2873	0.4562	0.4192	0.5666	0.5635
R8	0.6548	0.5356	0.4825	0.5763	0.3819	0.5747	0.5036	1.0000	0.3242	0.5292	0.2540	0.4034	0.3707	0.5927	0.5729
R9	0.4519	0.4517	0.4517	0.4517	0.4517	0.4518	0.4517	0.4518	1.0000	0.4518	0.3534	0.4517	0.4516	0.4518	0.4517
R10	0.5875	0.5684	0.5310	0.5812	0.4232	0.5860	0.5518	0.5866	0.3594	1.0000	0.2815	0.4471	0.4108	0.5835	0.5814
R11	0.3908	0.3907	0.3907	0.3907	0.3907	0.3907	0.3907	0.3907	0.3902	0.3907	1.0000	0.3907	0.3907	0.3907	0.3907
R12	0.5098	0.5079	0.5094	0.5095	0.4802	0.5095	0.5094	0.5095	0.4095	0.5096	0.3208	1.0000	0.4677	0.5095	0.5095
R13	0.4869	0.4866	0.4866	0.4867	0.4847	0.4866	0.4866	0.4867	0.4256	0.4866	0.3335	0.4862	1.0000	0.4867	0.4864
R14	0.6396	0.5487	0.4940	0.5858	0.3914	0.5745	0.5133	0.6073	0.3322	0.5394	0.2603	0.4133	0.3798	1.0000	0.5932
R15	0.6162	0.5621	0.5089	0.5994	0.4049	0.5862	0.5283	0.6076	0.3438	0.5563	0.2694	0.4278	0.3929	0.6139	1.0000

Table A.4: Purchases profile industries' subsethood measures with high levels above 0.5 highlighted

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
C1	1.0000	0.5378	0.4734	0.5949	0.4884	0.3846	0.4258	0.4614	0.4488	0.3687	0.4834	0.3710	0.4778	0.4160	0.5173
C2	0.6080	1.0000	0.5332	0.6068	0.5511	0.4346	0.4797	0.5201	0.5048	0.4168	0.5433	0.4193	0.5340	0.4702	0.5790
C3	0.5698	0.5676	1.0000	0.5697	0.5533	0.4626	0.5039	0.5502	0.5370	0.4436	0.5577	0.4464	0.5532	0.5003	0.5627
C4	0.6372	0.5749	0.5070	1.0000	0.5228	0.4118	0.4561	0.4936	0.4791	0.3949	0.5176	0.3974	0.5106	0.4455	0.5535
C5	0.5790	0.5780	0.5451	0.5788	1.0000	0.4558	0.5033	0.5365	0.5202	0.4370	0.5583	0.4397	0.5531	0.4923	0.5745
C6	0.5112	0.5110	0.5109	0.5111	0.5110	1.0000	0.5095	0.5106	0.5109	0.4846	0.5109	0.4859	0.5109	0.5103	0.5110
C7	0.5393	0.5373	0.5303	0.5393	0.5376	0.4854	1.0000	0.5292	0.5275	0.4667	0.5335	0.4684	0.5338	0.5187	0.5356
C8	0.5629	0.5612	0.5577	0.5623	0.5520	0.4685	0.5097	1.0000	0.5411	0.4497	0.5570	0.4526	0.5525	0.5047	0.5573
C9	0.5538	0.5510	0.5506	0.5520	0.5414	0.4743	0.5140	0.5474	1.0000	0.4548	0.5477	0.4578	0.5459	0.5123	0.5474
C10	0.4999	0.4998	0.4997	0.4998	0.4997	0.4942	0.4996	0.4997	0.4997	1.0000	0.4998	0.4885	0.4999	0.4990	0.4997
C11	0.5765	0.5731	0.5526	0.5763	0.5616	0.4584	0.5024	0.5445	0.5293	0.4397	1.0000	0.4424	0.5642	0.4959	0.5680
C12	0.5016	0.5015	0.5015	0.5016	0.5014	0.4942	0.5001	0.5016	0.5016	0.4872	0.5015	1.0000	0.5015	0.5004	0.5015
C13	0.5728	0.5663	0.5510	0.5715	0.5593	0.4608	0.5052	0.5429	0.5304	0.4420	0.5671	0.4446	1.0000	0.4985	0.5640
C14	0.5330	0.5328	0.5326	0.5329	0.5319	0.4918	0.5247	0.5301	0.5319	0.4716	0.5328	0.4741	0.5327	1.0000	0.5328
C15	0.5966	0.5906	0.5392	0.5960	0.5588	0.4433	0.4877	0.5269	0.5116	0.4251	0.5493	0.4278	0.5425	0.4796	1.0000

Table A.5: Sales profile between-clusters distance

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15
R1	0.0000	0.1731	0.1784	0.1680	0.1858	0.1689	0.1762	0.1587	0.1895	0.1735	0.1933	0.1845	0.1866	0.1615	0.1672
R2	0.1731	0.0000	0.2333	0.2284	0.2368	0.2287	0.2322	0.2238	0.2384	0.2310	0.2400	0.2362	0.2371	0.2254	0.2281
R3	0.1784	0.2333	0.0000	0.2310	0.2386	0.2312	0.2342	0.2264	0.2400	0.2332	0.2414	0.2380	0.2389	0.2282	0.2307
R4	0.1680	0.2284	0.2310	0.0000	0.2347	0.2260	0.2299	0.2208	0.2364	0.2284	0.2382	0.2340	0.2351	0.2226	0.2253
R5	0.1858	0.2368	0.2386	0.2347	0.0000	0.2349	0.2377	0.2308	0.2424	0.2367	0.2435	0.2408	0.2416	0.2321	0.2344
R6	0.1689	0.2287	0.2312	0.2260	0.2349	0.0000	0.2300	0.2210	0.2366	0.2287	0.2383	0.2342	0.2353	0.2231	0.2257
R7	0.1762	0.2322	0.2342	0.2299	0.2377	0.2300	0.0000	0.2250	0.2392	0.2320	0.2407	0.2371	0.2381	0.2270	0.2295
R8	0.1587	0.2238	0.2264	0.2208	0.2308	0.2210	0.2250	0.0000	0.2327	0.2237	0.2348	0.2300	0.2312	0.2177	0.2206
R9	0.1895	0.2384	0.2400	0.2364	0.2424	0.2366	0.2392	0.2327	0.0000	0.2382	0.2443	0.2419	0.2427	0.2341	0.2361
R10	0.1735	0.2310	0.2332	0.2284	0.2367	0.2287	0.2320	0.2237	0.2382	0.0000	0.2398	0.2359	0.2371	0.2255	0.2281
R11	0.1933	0.2400	0.2414	0.2382	0.2435	0.2383	0.2407	0.2348	0.2443	0.2398	0.0000	0.2431	0.2437	0.2360	0.2379
R12	0.1845	0.2362	0.2380	0.2340	0.2408	0.2342	0.2371	0.2300	0.2419	0.2359	0.2431	0.0000	0.2411	0.2315	0.2337
R13	0.1866	0.2371	0.2389	0.2351	0.2416	0.2353	0.2381	0.2312	0.2427	0.2371	0.2437	0.2411	0.0000	0.2326	0.2348
R14	0.1615	0.2254	0.2282	0.2226	0.2321	0.2231	0.2270	0.2177	0.2341	0.2255	0.2360	0.2315	0.2326	0.0000	0.2220
R15	0.1672	0.2281	0.2307	0.2253	0.2344	0.2257	0.2295	0.2206	0.2361	0.2281	0.2379	0.2337	0.2348	0.2220	0.0000

Table A.6: Purchases profile between-clusters distance

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
C1	0.0000	0.2139	0.2190	0.2084	0.2175	0.2238	0.2219	0.2193	0.2203	0.2247	0.2178	0.2246	0.2183	0.2222	0.2152
C2	0.2139	0.0000	0.2301	0.2221	0.2293	0.2341	0.2325	0.2304	0.2311	0.2346	0.2294	0.2345	0.2298	0.2328	0.2275
C3	0.2190	0.2301	0.0000	0.2260	0.2327	0.2369	0.2355	0.2335	0.2341	0.2373	0.2326	0.2372	0.2330	0.2357	0.2310
C4	0.2084	0.2221	0.2260	0.0000	0.2251	0.2304	0.2286	0.2264	0.2272	0.2310	0.2251	0.2308	0.2256	0.2289	0.2233
C5	0.2175	0.2293	0.2327	0.2251	0.0000	0.2362	0.2347	0.2330	0.2336	0.2368	0.2318	0.2366	0.2323	0.2350	0.2303
C6	0.2238	0.2341	0.2369	0.2304	0.2362	0.0000	0.2387	0.2371	0.2376	0.2403	0.2362	0.2402	0.2366	0.2389	0.2349
C7	0.2219	0.2325	0.2355	0.2286	0.2347	0.2387	0.0000	0.2357	0.2363	0.2391	0.2348	0.2390	0.2351	0.2376	0.2334
C8	0.2193	0.2304	0.2335	0.2264	0.2330	0.2371	0.2357	0.0000	0.2342	0.2375	0.2329	0.2373	0.2333	0.2359	0.2314
C9	0.2203	0.2311	0.2341	0.2272	0.2336	0.2376	0.2363	0.2342	0.0000	0.2380	0.2334	0.2378	0.2339	0.2364	0.2320
C10	0.2247	0.2346	0.2373	0.2310	0.2368	0.2403	0.2391	0.2375	0.2380	0.0000	0.2366	0.2405	0.2369	0.2393	0.2354
C11	0.2178	0.2294	0.2326	0.2251	0.2318	0.2362	0.2348	0.2329	0.2334	0.2366	0.0000	0.2365	0.2321	0.2350	0.2303
C12	0.2246	0.2345	0.2372	0.2308	0.2366	0.2402	0.2390	0.2373	0.2378	0.2405	0.2365	0.0000	0.2369	0.2392	0.2352
C13	0.2183	0.2298	0.2330	0.2256	0.2323	0.2366	0.2351	0.2333	0.2339	0.2369	0.2321	0.2369	0.0000	0.2354	0.2308
C14	0.2222	0.2328	0.2357	0.2289	0.2350	0.2389	0.2376	0.2359	0.2364	0.2393	0.2350	0.2392	0.2354	0.0000	0.2336
C15	0.2152	0.2275	0.2310	0.2233	0.2303	0.2349	0.2334	0.2314	0.2320	0.2354	0.2303	0.2352	0.2308	0.2336	0.0000

Table A.7: Industries' and clusters' entropies

	Industries' Entropy		Cluster's Entropy	
Ind.	**Sale**	**Purchase**		
1	2.6507	2.6915	**Sale**	
2	0.3706	0.4567	**R1**	3.0517
3	2.6587	2.6824	**R2**	2.4837
4	0.2785	0.3239	**R3**	2.4030
5	2.6759	2.6991	**R4**	2.5452
6	0.4693	0.3527	**R5**	2.2160
7	2.6514	2.6924	**R6**	2.5403
8	0.1418	0.1765	**R7**	2.4429
9	2.6299	2.6941	**R8**	2.6337
10	0.4591	0.2363	**R9**	2.0766
11	2.6588	2.6968	**R10**	2.4796
12	0.3241	0.3126	**R11**	1.8651
13	2.6773	2.6994	**R12**	2.2590
14	0.6526	0.2890	**R13**	2.1896
15	2.6447	2.6908	**R14**	2.5988
16	0.0878	0.1590	**R15**	2.5480
17	2.6562	2.6961	**Purchase**	
18	0.3717	0.3541		
19	2.7068	2.7037	**C1**	2.6954
20	0.0416	0.1645	**C2**	2.5156
21	2.6832	2.6981	**C3**	2.4176
22	0.1679	0.3382	**C4**	2.6008
23	2.6647	2.6954	**C5**	2.4426
24	0.1285	0.2252	**C6**	2.2527
25	2.6289	2.6880	**C7**	2.3317
26	0.5874	0.6225	**C8**	2.4016
27	2.5698	2.6426	**C9**	2.3804
28	0.4831	0.4183	**C10**	2.2196
29	2.6077	2.6872	**C11**	2.4382
30	2.6235	2.6981	**C12**	2.2256
31	2.6650	2.6951	**C13**	2.4271
32	2.5485	2.6429	**C14**	2.3172
33	2.5883	0.9331	**C15**	2.4867

Table A.8: Fuzzy Shapley values (%)

Ind.	Sales	Ind.	Purchases
20	3.1269	**20**	3.1227
16	3.1250	**16**	3.1225
24	3.1228	**8**	3.1213
8	3.1221	**24**	3.1191
22	3.1208	**10**	3.1187
4	3.1131	**14**	3.1165
12	3.1098	**12**	3.1148
18	3.1056	**4**	3.1141
2	3.1040	**22**	3.1126
10	3.0963	**18**	3.1118
6	3.0944	**6**	3.1106
28	3.0930	**28**	3.1055
26	3.0798	**2**	3.1028
14	3.0728	**26**	3.0905
19	2.8865	**19**	2.8922
13	2.8082	**13**	2.8850
21	2.8064	**5**	2.8715
5	2.7914	**21**	2.8667
23	2.7497	**17**	2.8589
11	2.7449	**11**	2.8568
31	2.7438	**15**	2.8510
17	2.7346	**9**	2.8493
3	2.7289	**23**	2.8425
7	2.7172	**30**	2.8421
1	2.7151	**7**	2.8398
15	2.7051	**31**	2.8351
9	2.6516	**1**	2.8341
25	2.6504	**3**	2.8149
30	2.6365	**25**	2.8060
29	2.5977	**29**	2.7908
33	2.5567	**27**	2.6401
27	2.5187	**32**	2.6295
32	2.4783	**33**	0.5561

REFERENCES

Bergman, E. M. and E. J. Feser (1999) Industry Clusters: A Methodology and Framework for Regional Development Policy in the United States, in Reolandt, T. and P. den Hertog (eds.) *Boosting Innovation: The Cluster Approach*, Organization for Economic Cooperation and Development, Paris, Ch. 10: 243-268.

Cella, G. (1984) The Input-Output Measurement of Interindustry Linkages, *Oxford Bulletin of Economics and Statistics* 46: 73-84.

Cella, G. (1986) The Input-Output Measurement of Interindustry Linkages: A Reply, *Oxford Bulletin of Economics and Statistics* 48: 379-384.

Clements, B. J. (1990) On the Decomposition and Normalization of Interindustry Linkages, *Economics Letters* 33: 337-340.

Dridi, C. and G. J.D. Hewings (2002a) Sectors Associations and Similarities in Input-Output Systems: An Application of Dual Scaling and Fuzzy Logic to Canada and the United States, Discussion Paper REAL 01-T-15, *Regional Economics Applications Laboratory, University of Illinois, Urbana* (forthcoming in *The Annals of Regional Science*).

Dridi, C. and G. J.D. Hewings (2002b) Toward a Quantitative Analysis of Industrial Clusters I: Fuzzy Cluster vs. Crisp Clusters, *Technical Papers Series* REAL 02-T-1, *Regional Economics Applications Laboratory*, University of Illinois, Urbana.

Dumitrescu, D., B. Lazzerini, and L. C. Jain (2000) *Fuzzy Sets and Their Application to Clustering and Training*, CRC Press, Boca Raton, Florida.

Feser, E. J. (1998) Old and New Theories of Industry Clusters, in Steiner, M. (ed.) *Clusters and Regional Specialization*, Pion, London: 19-40.

Feser, E. J. and E. M. Bergman (2000) National Industry Clusters Templates: A Framework for Applied Regional Clusters Analysis, *Regional Studies* 34, 1: 1-19.

Feser, E. J. and M. I. Luger (2001) Cluster Analysis as a Mode of Inquiry: Its Use in Science and Technology Policymaking in North Carolina, *Manuscript, University of North Carolina at Chapel Hill*.

Feser, E. J. and S. H. Sweeney (2000) A Test for the Coincident Economic and Spatial Clustering of Business Enterprises, *Journal of Geographical Systems* 2, 4: 349-373.

Feser, E. J., K. Koo, H. C. Renski, and S. H. Sweeney (2001) Incorporating Spatial Analysis in Applied Industry Cluster Studies, Manuscript, Department of City and Regional Planning, University of North Carolina, Chapel Hill, North Carolina.

Guilhoto, J.J.M, M. Sonis, G. J.D. Hewings (1994) Using input-output to measure interindustry linkages: a new perspective, *Technical Papers Series REAL 94-T-9, Regional Economics Applications Laboratory*, University of Illinois, Urbana.

Kaufman, L. and P. J. Rousseeuw (1990) *Finding Groups in Data: An Introduction to Cluster Analysis*, Wiley, New York, Ch. 4: 164-198.

Lootsma, F. A. (1997) *Fuzzy Logic for Planning and Decision Making*, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nishisato, S. (1980) *Analysis of Categorical Data: Dual Scaling and its Application*, University of Toronto Press, Toronto.

Nishisato, S. (1994) *Elements of Dual Scaling: An Introduction to Practical Data Analysis*, Lawrence Erlbaum Associates, New Jersey.

Parr, J.B., G. J.D. Hewings, J. Sohn, and S. Nazara (2002) Agglomeration and trade: some additional perspectives, *Regional Studies* 36: 675-684.

Porter, M. E. (1990) *The Competitive Advantage of Nations*, The Free Press, New York.

Porter, M. E. (1998) "Clusters and the New Economics of Competition", *Harvard Business Review* (November-December): 77-90.

Reza, F. M. (1961) *An Introduction to Information Theory*, McGraw-Hill, New York, Ch. 3, 76-130.

Seo, John J.Y. and G.J.D. Hewings. (2003) The Impact of International Exports on the Midwest Economies, *Discussion Paper, Regional Economics Applications Laboratory*, University of Illinois, Urbana (forthcoming).

Shapley, L. S. (1953) A Value for *n*-Person Games", in H. Kuhn (ed., 1997), *Classics in Game* Theory, Princeton University Press, Princeton, New Jersey, 69-79.

Shubik, M. (1985) *Game Theory in the Social Science: Concepts and Solutions*, The MIT Press, Cambridge, Massachusetts.

Sonis M. and G. J.D. Hewings (1992) Coefficient change in input-output models: theory and applications, *Economic Systems Research* 4: 143-157.

Sonis, M., G. J.D. Hewings and J. Guo (2000) A New Image of Classical Key Sector Analysis: Minimum Information Decomposition of the Leontief Inverse, *Economic Systems Research* 12: 401-423

Ward, J. H. Jr (1963) Hierarchical Grouping to Optimize an Objective Function, *Journal of the American Statistical Association* 58, 301: 236-244.

Yager, R. R. (2000) On the Entropy of Fuzzy Measures, *IEEE Transactions Fuzzy Systems* 8, 4: 453-461.

Yager, R. R. (2001) Characterizing Fuzzy Measures Used in Uncertainty Representation, *2nd International Symposium on Imprecise Probabilities and Their Applications*, Ithaca, New York.

Zadeh, L. A. (1965) Fuzzy Sets, *Information and Control* 8, 3: 338-353.

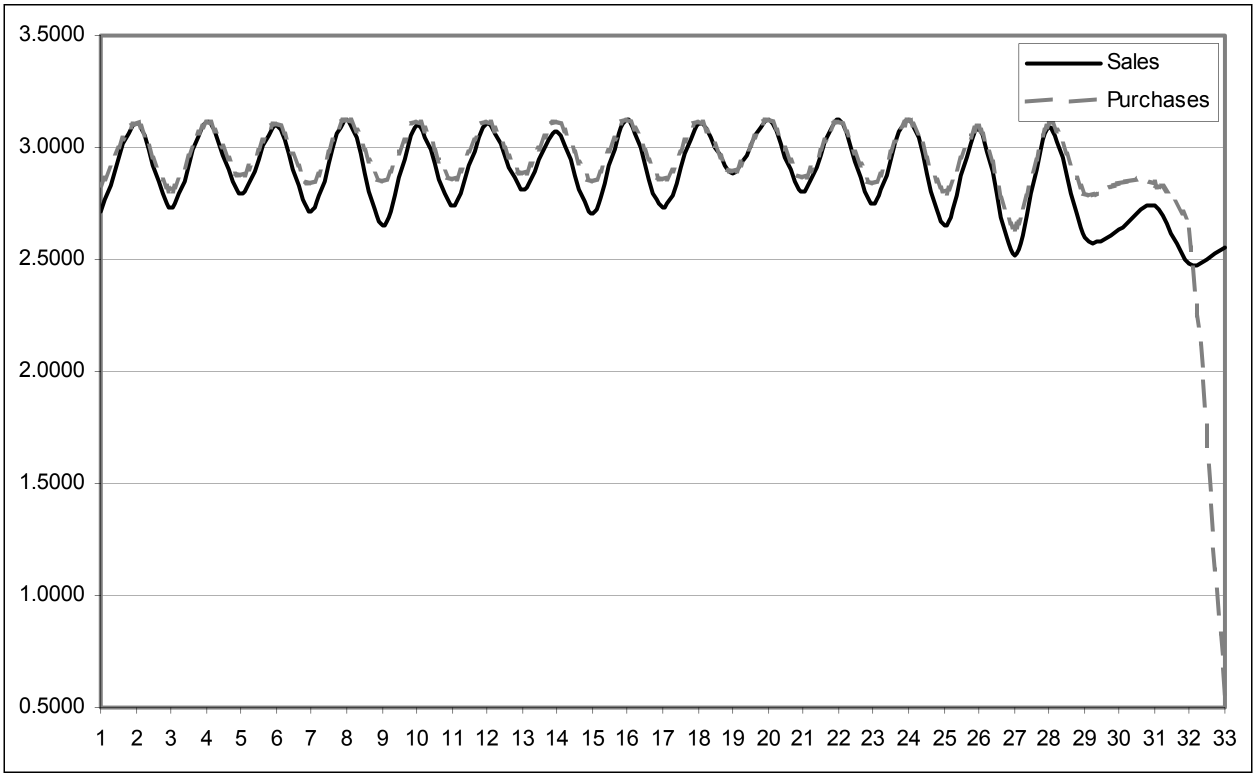

Figure 1: Fuzzy Shapley value for in the sales and purchases profiles (in %)