MIYAZAWA MEET CHRISTALLER: SPATIAL MULTIPLIERS WITHIN A TRIPLE DECOMPOSITION OF INPUT-OUTPUT CENTRAL PLACE SYSTEM

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Miyazawa meet Christaller: Spatial Multipliers within a Triple Decomposition of Input-output Central Place Systems

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ABSTRACT: A new triple *UD*L-factorization (*U* represents an upper-triangular block matrix, *L* represents a lower-triangular block matrix, and *D* represents the diagonal block matrix) of the Leontief inverse is based on the Schur block-inversion of matrices. This factorization is applied to the decomposition of the Leontief inverse for input-output systems within a central place hierarchy of the Christaller-Lösch, Beckmann-McPherson type. Such a factorization reflects the process of gradual complication of the central place hierarchy and the parallel augmentation of backward and forward linkages within it. In this scheme the classical Miyazawa interrelation income multipliers play the role of spatial backward and forward linkages multipliers within developing hierarchical central place systems of towns, cities and central capital.

1. Introduction: The problem posed.

The exploration of the space-time evolution of economies has gathered momentum with the introduction of nonlinear relative dynamics and consideration of catastrophe and bifurcation properties of spatial systems. However, there has been surprisingly little effort directed towards the exploration of linkages between two well-known structural models – the input-output system that embraces interdependence between sectors and the central place system that addresses the spatial structure of settlements. However, the idea to investigate the input-output relationship within the central place system is not a new one. The necessity to combine together the hierarchical structure of central place system with the input-output structure of the transaction flows within one unifying framework was stressed in the programmatic paper of Isard (1960, p.141). The next attempt to mention some ideas of central place theory useful for the description of the regional economic interaction was made by Chalmers *et al.,* 1978; however, no attention was directed to the intricate hierarchical structure of central place systems. A more systematic treatment of this problem was undertaken by Robison and Miller (1991). They used the rudimentary structure of (capital and neighboring cities) an intercommunity central place system, without paying attention to the fine structure of the central place hierarchy. The complexity of mathematical presentation was limited to the level of a simple two-community two-order sub region level with one dominant central place.

In this paper, an attempt will be made to integrate the theory of central place hierarchies and multi-regional input-output analysis. In central place theory, more attention is placed on the structure of consumption than on production, creating an opportunity to explore the way in which interindustrial interdependence serves to complicate the hierarchy of central places. Hence, it will be possible to show in what way the decomposition of the Leontief inverse for input-output central place systems reflects the process of complication of the evolving hierarchy of central places. To provide a more comprehensive picture of the structure of interdependence, the Leontief system will be augmented to include consideration of income generation and consumption using the framework proposed by Miyazawa (1976). An important contribution of Miyazawa's system is the specification of a matrix of interrelational income multipliers; these multipliers explore the ways in which income creation in one group or region generates income in other parts of the system. In a central place system, the focus will be on the spatial structure of the economy. The main feature of the discussion will be on the interpretation of the classical Miyazawa interrelational multipliers as *spatial* multipliers in the developing structure of backward and forward linkages within a three-level hierarchical central place systems of towns, cities and a central capital.

In the next section, the description of the central place system will be provided. Section 3 will apply a triangular decomposition of the Leontief inverse for an input-output based central place system. Sections 4 and 5 provide the decomposition for a top-down and a bottom-up system. Section 6 introduced the Miyazawa income-consumption distribution framework that is then integrated into the central place system in section 7. The fine structure of this distribution is revealed in section 8 and the paper concludes with some reflections on challenges for empirical implementation.

2. Structure of the Classical Central Place System

The spatial description of the original Christaller central place model is based on three generic geometric properties of central places associated this central place system (see Sonis, 1986):

1. The first property is that all hinterland areas of the central places at the same hierarchical level form a hexagonal covering of the plane with the centers on the homogeneous triangular lattice;

2. The second property is that the size of the hinterland areas increases from the smallest (on the lower tier of the central place hierarchy) to the largest (on the highest tier of hierarchy) by a constant nesting factor *k*. This nesting factor expresses one of the Christaller's three principles, namely, marketing (*k*=3), transportation (*k*=4) and administrative (*k*=7) principles;

3. The third property is that the center of a hinterland area of a given size is also the center of an hinterland of each smaller size (Christaller, 1933, Sonis, 1985)

The Beckmann-McPherson (1970) central place model differs from the Christaller framework by applying variable nesting factors and by using the Löshian principle of all possible coverings of the plane by hexagons of variable integer sizes. Their centers are the vertices of the initial Christaller triangular lattice (Lösch, 1940).

In this paper, a stylized example of the Beckmann-McPherson three tier central place system will be used, including an urban hierarchy with a single largest central place K and two further hierarchical levels, cities, $C=\{C_1,C_2,C_3,C_4,C_5,C_6\}$, and towns $T=\{T_1,T_2,T_3,T_4,T_5,T_6\}$ (see figure 1). Finally, the system is assumed to be for a closed economy with just one complete hierarchy; hence, there is no external trade.

<<insert Figure 1 here>>

3. The Application of the Triangular Decomposition of the Leontief Inverse for the Analysis of the Input-Output Central Place System

To begin, the central place spatial organization of settlements is set aside and only the hierarchy and the flow of intermediate goods between these three hierarchical levels will be considered. In such a case, the matrix of direct inputs can be presented in the form:

$$A = \begin{bmatrix} A_{TT} & A_{TC} & A_{TK} \\ A_{CT} & A_{CC} & A_{CK} \\ A_{KT} & A_{KC} & A_{KK} \end{bmatrix} \tag{1}$$

For the analysis of such an input-output system, the following structure of the Leontief inverse $B = (I - A)^{-1}$ will be used together with the a triangular decomposition (*cf* Sonis and Hewings, 2000):

$$B = (I - A)^{-1} = \begin{bmatrix} I & & \\ B^2_{CC}(C,K)A^3_{CT} & I & \\ B^2_{KK}(C,K)A^3_{KT} & B_K A_{KC} & I \end{bmatrix} \begin{bmatrix} B^3_{TT} & & \\ & B^2_{CC}(C,K) & \\ & & B_K \end{bmatrix} \begin{bmatrix} I & A^3_{TC}B^2_{CC}(C,K) & A^3_{TK}B^2_{KK}(C,K) \\ & I & A_{CK}B_K \\ & & I \end{bmatrix} \tag{2}$$

In equation (2), the following definitions of components are used:

1) The augmented inputs[1] representing the coinfluence of different hierarchical levels (see Sonis and Hewings, 1998) have a form:

$$\begin{aligned} A^3_{TC} &= A_{TC} + A_{TK}B_K A_{KC} \\ A^3_{CT} &= A_{CT} + A_{CK}B_K A_{KT} \\ A^3_{KT} &= A_{KT} + A_{KC}B_C A_{CT} \\ A^3_{TK} &= A_{TK} + A_{TC}B_C A_{CK} \end{aligned} \tag{3}$$

2) The Leontief inverses of the economies on the three separate hierarchical levels of the Beckmann-McPherson Central place system are

$$B_T = (I - A_{TT}); B_C = (I - A_{CC}); B_K = (I - A_{KK}) \tag{4}$$

3) The extended Leontief inverses of the hierarchical level of intermediate cities C under the influence of the central city K and of central city K under influence of intermediate cities C are

$$B^2_{CC}(K,C) = [I - A_{CC} - A_{CK}B_K A_{KC}]^{-1}; B^2_{KK}(K,C) = [I - A_{KK} - A_{KC}B_C A_{CK}]^{-1} \tag{5}$$

[1] The term, augmentated inputs, was first introduced by Yamada and Ihara (1969) in their discussion of interregional feedback effects. For further discussion, see Sonis and Hewings (2001).

4) The component

$$B_{TT}^3 = \left[I - A_{TT} - A_{TC} B_{CC}^2 (K,C) A_{CT}^3 - A_{TK} B_{KK}^2 (K,C) A_{KT}^3 \right]^{-1} \tag{6}$$

represents the extended Leontief inverse of the hierarchical level of towns, *T,* under the influence of the central city *K* and the intermediate cities *C*.

Further, the input-output partial central place system, including only two hierarchical levels of central city *K* and intermediate cities *C* and not including the level of towns *T,* corresponds to the block matrix of direct inputs

$$A(K,C) = \begin{bmatrix} A_{CC} & A_{CK} \\ A_{KC} & A_{KK} \end{bmatrix} \tag{7}$$

Its Leontief inverse has a form

$$B(K,C) = \left[I - A(K,C) \right]^{-1} = \begin{bmatrix} B_{CC}^2 (K,C) & B_{CC}^2 (K,C) A_{CK} B_K \\ B_{KK}^2 (K,C) A_{KC} B_C & B_{KK}^2 (K,C) \end{bmatrix} \tag{8}$$

Its the triangular decomposition (see Sonis and Hewings, 2000, p.571) is

$$\begin{aligned} B(K,C) &= \begin{bmatrix} I & \\ B_K A_{KC} & I \end{bmatrix} \begin{bmatrix} B_{CC}^2 (K,C) & \\ & B_K \end{bmatrix} \begin{bmatrix} I & A_{CK} B_K \\ & I \end{bmatrix} = \\ &= \begin{bmatrix} I & \\ L_{KC} & I \end{bmatrix} \begin{bmatrix} B_{CC}^2 (K,C) & \\ & B_K \end{bmatrix} \begin{bmatrix} I & U_{CK} \\ & I \end{bmatrix} \end{aligned} \tag{9}$$

The backward and forward linkages L_{KC}, U_{CK} together generate the feedback loop of the form:

$$K \to C \to K$$

In the triangular decomposition (2) the lower triangular matrix

$$L = \begin{bmatrix} I & & \\ B_{CC}^2 (T) A_{CT}^3 & I & \\ B_{KK}^2 (T) A_{KT}^3 & B_K A_{KC} & I \end{bmatrix} = \begin{bmatrix} I & & \\ L_{CT} & I & \\ L_{KT} & L_{KC} & I \end{bmatrix} \tag{10}$$

represents the backward linkages of all three hierarchical levels such that the second block column of L:

$$\begin{bmatrix} 0 \\ I \\ B_K A_{KC} \end{bmatrix} = \begin{bmatrix} 0 \\ I \\ L_{KC} \end{bmatrix}$$

represents the new backward linkages of the subsystem of two hierarchical levels of central city K and intermediate cities C appearing as a result of adding to the economics of central city K the economics of intermediate cities C; the first column of the matrix L:

$$\begin{bmatrix} I \\ B_{CC}^2(T) A_{CT}^3 \\ B_{KK}^2(T) A_{KT}^3 \end{bmatrix} = \begin{bmatrix} I \\ L_{CT} \\ L_{KT} \end{bmatrix}$$

represents the extension of the backward linkages within the system C appearing as a result of further extension of to the economics of central city K and intermediate cities C with the help of adding the economy of hierarchical level of the towns T.

Analogously, the upper triangular matrix

$$U = \begin{bmatrix} I & A_{TC}^3 B_{CC}^2(T) & A_{TK}^3 B_{KK}^2(T) \\ & I & A_{CK} B_K \\ & & I \end{bmatrix} = \begin{bmatrix} I & U_{TC} & U_{TK} \\ & I & U_{CK} \\ & & I \end{bmatrix} \qquad (11)$$

represents the growing system of forward linkages within the augmented system of all three hierarchical levels.

The backward and forward augmentation of linkages L_{CT}, U_{TC}; L_{KT}, U_{TK}; and L_{KC}, U_{CK} present three feedback loops of spatial economic dependencies:

$$\begin{array}{l} C \to T \to C \\ K \to T \to K \\ K \to C \to K \end{array}$$

The diagonal block matrix

$$D = \begin{bmatrix} B_{TT}^{3} & & \\ & B_{CC}^{2}(T) & \\ & & B_{K} \end{bmatrix} \tag{12}$$

represents the Leontief inverse of the hierarchical level of the central city *K* and the extended Leontief inverses corresponding to the augmentation of the level *K* to the two hierarchical levels *K, C* and to thence to the three hierarchical levels *K, C, T*.

The augmentation of the backward and forward linkages in a three-tier hierarchical system presents the stages of development of the Central Place system starting from one central place (capital) and gradually expending to two hierarchical levels (capital and neighboring cities) and further, to three hierarchical levels (capital, neighboring cities and surrounding towns)

In the two following sections, the decomposition will be presented for a top-down and bottom up central place model respectively.

4. Decomposition of Leontief Inverse for Input-Output "Top-down" Central Place Model

If the forward linkages in the system *K, C, T* are negligible, then the matrix of direct inputs will have the form

$$A = \begin{bmatrix} A_{TT} & 0 & 0 \\ A_{CT} & A_{CC} & 0 \\ A_{KT} & A_{KC} & A_{KK} \end{bmatrix} \tag{13}$$

then the triangular representation (2) will include only backward linkages:

$$B = \begin{bmatrix} I & & \\ B_C A_{CT} & I & \\ B_K(A_{KT} + A_{KC} B_C A_{CT}) & B_K A_{KC} & I \end{bmatrix} \begin{bmatrix} B_T & & \\ & B_C & \\ & & B_K \end{bmatrix} \tag{14}$$

This decomposition corresponds to the circulation of flows of intermediate flows within the central places and to the case of "top-down" transaction flows where each central place includes the same structure of industries and is sending "transaction flows" of direct inputs to each dependent central place within its own hinterland area and exchanging production with the central places of the same hierarchical tier. In a sense, this production structure mimics the strict hierarchical consumption structure assumed for the central place system.

5. Decomposition of the Leontief Inverse for Input-Output "Bottom-up" Central Place Model

Of course, there exists a polar opposite case, the "bottom-up" system of transaction flows in which there are negligible backward linkages with the following block-matrix of direct inputs:

$$A = \begin{bmatrix} A_{TT} & A_{TC} & A_{TK} \\ 0 & A_{CC} & A_{CK} \\ 0 & 0 & A_{KK} \end{bmatrix} \tag{15}$$

The corresponding triangular decomposition of the Leontief inverse will include only forward linkages:

$$B = \begin{bmatrix} B_T & & \\ & B_C & \\ & & B_K \end{bmatrix} \begin{bmatrix} I & (A_{TC} + A_{TK} B_K A_{KC}) B_C & A_{TK} B_K \\ & I & A_{CK} B_K \\ & & I \end{bmatrix} \tag{16}$$

The block-structure form of the triangular decomposition (2) allows the incorporation of the fine structure of the spatial economic dependencies between central places within different hierarchical levels. Each such specification of this fine structure can be incorporated into the triangular decomposition (2).

Thus far, attention has only been directed to the production side of the economy. In the next section, consideration will be given to the income generated from production and the impact of expenditures from these incomes on goods and services produced in the system. While exchange in production need not be hierarchical, it will be assumed that consumption of goods is made according to the usual central place principles of seeking the good at the nearest n the hierarchy that offers it. Rather than adopting the usual closed form of the Leontief model in which households and income are made endogenous, an alternative specification based on the work of Miyazawa will be used to generate the structure of income-consumption linkages.

6. Miyazawa Income-Consumption Distribution in the Input-Output Model

Miyazawa (1968, and 1976 for the most complete exposition) introduced the matrix model:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} A & C \\ V & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} f \\ g \end{bmatrix} \qquad (17)$$

with a block matrix $M = \begin{bmatrix} A & C \\ V & 0 \end{bmatrix}$ for the analysis of the interrelationships among various income groups in the process of income formation. In the Miyazawa interpretation, the matrix A represents the inter industry direct inputs; vector X is gross output, vector f is final demand excluding consumption expenditures), vector Y represents the total income, the matrix V represents the value-added ratios; and the matrix C represents the coefficients of consumption expenditures. In addition to working with the usual Leontief interindustry inverse $B = (I - A)^{-1}$; Miyazawa introduced the matrix multiplier VB showing the induced income earned from production activities among industries; the matrix multiplier BC showing the induced production due to endogenous consumption (per unit of income) in each household sector; and the matrix multiplier $L = VBC$ showing interrelationships among incomes through the process of propagation from consumption expenditures.

The matrix $K = (I - L)^{-1} = (I - VBC)^{-1}$ is interpreted as the interrelational income multiplier, and the matrix multiplier KVB is interpreted as the matrix multiplier of income formation.

Further, the following Frobenius-Schur form:

$$B_e = (I - A - CV)^{-1} \qquad (18)$$

may be referred to as the enlarged or augmented Leontief inverse, with the properties:

$$KVB = VB_e; \quad BCK = B_eC \qquad (19)$$

For the Miyazawa income generation scheme based on block matrix $M = \begin{bmatrix} A & C \\ V & 0 \end{bmatrix}$, the Leontief block-inverse has the following decomposition that separates the backward and forward linkages effects:

$$B(M) = (I - M)^{-1} = \begin{bmatrix} B_e & B_e C \\ VB_e & K \end{bmatrix} = \begin{bmatrix} B_e & BCK \\ KVB & K \end{bmatrix} \qquad (20)$$

The structure of the interconnection between the interrelational income multiplier, *K*, and the enlarged Leontief inverse B_e can be seen and interpreted from the following formula:

$$K = I + VB_e C \qquad (21)$$

where the enlarged matrix multiplier $L_e = VB_e C$ shows the interrelationships among incomes through the process of enhanced propagation from consumption expenditures.

Next, the fundamental *UDL*-factorization

$$B(M) = (I - M)^{-1} = \begin{bmatrix} B_e & BCK \\ KVB & K \end{bmatrix} = \begin{bmatrix} I & BC \\ 0 & I \end{bmatrix} \begin{bmatrix} B & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} I & 0 \\ VB & I \end{bmatrix} \qquad (22)$$

separates multiplicatively the induced income earned from production activities among industries, *VB*, and the induced production due to endogenous consumption (per unit of income) in each household sector *BC* from the industrial production activities. In this factorization, the interrelational income multiplier, *K,* appears explicitly and the enlarged Leontief inverse, B_e, appears implicitly. The structure of the interconnection between the enlarged Leontief inverse, B_e, and the interrelational income multiplier, *K,* can be seen and interpreted from the following formula:

$$B_e = B + BCKVB \qquad (23)$$

7. Miyazawa Income-Consumption Distribution in Input-Output Central Place Model

The Miyazawa income-consumption distribution in the input-output hierarchical central place model can be presented with the help of the following matrix:

$$M = \begin{bmatrix} A_{TT} & A_{TC} & A_{TK} & C_T \\ A_{CT} & A_{CC} & A_{CK} & C_C \\ A_{KT} & A_{KC} & A_{KK} & C_K \\ V_T & V_C & V_K & \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{C} \\ \tilde{V} & \end{bmatrix} \tag{24}$$

where the matrix $\tilde{A}$ is the block matrix of direct inputs for the different hierarchical levels of towns, cities and capital city; $\tilde{V} = \begin{pmatrix} V_T & V_C & V_K \end{pmatrix}$ and $\tilde{C} = \begin{pmatrix} C_T \\ C_C \\ C_K \end{pmatrix}$ represent the income and consumption respectively, corresponding to the same spatial levels in the central place hierarchy. Analogously to (2), the following expression provides the following triple decomposition:

$$\begin{aligned} B(M) &= \\ &= \begin{bmatrix} I & & & \\ & I & & \\ & & I & \\ V_T & V_C & V_K & I \end{bmatrix} \begin{bmatrix} I & & & \\ L_{CT} & I & & \\ L_{KT} & L_{KC} & I & \\ & & & I \end{bmatrix} \begin{bmatrix} D_T & & & \\ & D_C & & \\ & & D_K & \\ & & & I \end{bmatrix} \begin{bmatrix} I & U_{TC} & U_{TK} & \\ & I & U_{CK} & \\ & & I & \\ & & & I \end{bmatrix} \begin{bmatrix} I & & & C_T \\ & I & & C_C \\ & & I & C_K \\ & & & I \end{bmatrix} = \\ &= \begin{bmatrix} L & \\ \tilde{V}L & I \end{bmatrix} \begin{bmatrix} D & \\ & I \end{bmatrix} \begin{bmatrix} U & U\tilde{C} \\ & I \end{bmatrix} \end{aligned} \tag{25}$$

where induced income and induced production are

$$\begin{aligned} \tilde{V}L &= \begin{pmatrix} V_T & V_C & V_K \end{pmatrix} \begin{bmatrix} I & & \\ L_{CT} & I & \\ L_{KT} & L_{KC} & I \end{bmatrix} = \begin{pmatrix} V_T + V_C L_{CT} + V_K L_{KT} & V_C + V_K L_{KT} & V_K \end{pmatrix} \\ U\tilde{C} &= \begin{bmatrix} I & & \\ U_{CT} & I & \\ U_{KT} & U_{KC} & I \end{bmatrix} \begin{bmatrix} C_T \\ C_C \\ C_K \end{bmatrix} = \begin{bmatrix} C_T \\ U_{CT} C_T + C_C \\ U_{KT} C_T + U_{KC} C_C + C_K \end{bmatrix} \end{aligned} \tag{26}$$

Therefore, the Miyazawa interrelational income multiplier (see 23) will have the form:

$$K = I + \begin{pmatrix} V_T & V_C & V_K \end{pmatrix} \begin{bmatrix} I & & \\ L_{CT} & I & \\ L_{KT} & L_{KC} & I \end{bmatrix} \begin{bmatrix} D_T & & \\ & D_C & \\ & & D_K \end{bmatrix} \begin{bmatrix} I & & \\ U_{CT} & I & \\ U_{KT} & U_{KC} & I \end{bmatrix} \begin{bmatrix} C_T \\ C_C \\ C_K \end{bmatrix} = \\ = \begin{pmatrix} V_T + V_C L_{CT} + V_K L_{KT} & V_C + V_K L_{KT} & V_K \end{pmatrix} \begin{bmatrix} D_T & & \\ & D_C & \\ & & D_K \end{bmatrix} \begin{bmatrix} C_T \\ U_{CT} C_T + C_C \\ U_{KT} C_T + U_{KC} C_C + C_K \end{bmatrix} \quad (27)$$

Analogously to the spatial feedback loop interpretation developed for (2), the Miyazawa interrelational income multiplier can be interpreted as a spatial multiplier in the central place input-output system. The fine structure of this system will be revealed in the next section.

8. The Fine Structure of the Miyazawa Income-Consumption Distribution in Input-Output Central Place Model

A simple case of the structure will be presented that includes the circulation of flows of intermediate goods within the central places and the "top-down" structure of the transaction flows between the dependent central places only. Such a structure is presented in figure 2.

<<insert figure 2 here>>

This scheme generalizes slightly the usual assumption of classical central place theory requiring that both income earned from production and consumption expenditures from this income will be concentrated in same location. The corresponding block-matrix of direct inputs reflecting the central place hierarchy of such system has a form:

$$A = \begin{bmatrix} A_{TT} & A_{TC} & A_{TK} \\ A_{CT} & A_{CC} & A_{CK} \\ A_{KT} & A_{KC} & A_{KK} \end{bmatrix} =$$

$$= \begin{bmatrix}
a_{T_1T_1} & & & & & & & & & & & & \\
& a_{T_2T_2} & & & & & & & & & & & \\
& & a_{T_3T_3} & & & & & & & & & & \\
& & & a_{T_4T_4} & & & & & & & & & \\
& & & & a_{T_5T_5} & & & & & & & & \\
& & & & & a_{T_6T_6} & & & & & & & \\
a_{C_1T_1} & & & & & a_{C_1T_6} & a_{C_1C_1} & & & & & & \\
a_{C_2T_1} & a_{C_2T_2} & & & & & & a_{C_2C_2} & & & & & \\
& a_{C_3T_2} & a_{C_3T_3} & & & & & & a_{C_3C_3} & & & & \\
& & a_{C_4T_3} & a_{C_4T_4} & & & & & & a_{C_4C_4} & & & \\
& & & a_{C_5T_4} & a_{C_5T_5} & & & & & & a_{C_5C_5} & & \\
& & & & a_{C_6T_5} & a_{C_6T_6} & & & & & & a_{C_6C_6} & \\
a_{KT_1} & a_{KT_2} & a_{KT_3} & a_{KT_4} & a_{KT_5} & a_{KT_6} & a_{KC_1} & a_{KC_2} & a_{KC_3} & a_{KC_4} & a_{KC_5} & a_{KC_6} & A_{KK}
\end{bmatrix} \quad (28)$$

The *LD*-substructure of the Leontief inverse for this input-output central place system has a form drawing on (14) that is as follows:

$$LD = \begin{bmatrix} I & & \\ B_C A_{CT} & I & \\ B_K (A_{KT} + A_{KC} B_C A_{CT}) & B_K A_{KC} & I \end{bmatrix} \begin{bmatrix} B_T & & \\ & B_C & \\ & & B_K \end{bmatrix}$$

where

$$B_T = \begin{bmatrix}
b_{T_1T_1} & & & & & \\
& b_{T_2T_2} & & & & \\
& & b_{T_3T_3} & & & \\
& & & b_{T_4T_4} & & \\
& & & & b_{T_5T_5} & \\
& & & & & b_{T_6T_6}
\end{bmatrix}, \quad (29)$$

$$B_C = \begin{bmatrix} b_{C_1C_1} & & & & & \\ & b_{C_2C_2} & & & & \\ & & b_{C_3C_3} & & & \\ & & & b_{C_4C_4} & & \\ & & & & b_{C_5C_5} & \\ & & & & & b_{C_6C_6} \end{bmatrix} \tag{30}$$

and where

$$b_{T_iT_i} = \left(I - a_{T_iT_i}\right)^{-1}; b_{C_iC_i} = \left(I - a_{C_iC_i}\right)^{-1}; i = 1, 2, ..., 6 \tag{31}$$

and

$$B_C A_{CT} = \begin{bmatrix} b_{C_1C_1}a_{C_1T_1} & & & & & b_{C_1C_1}a_{C_1T_1} \\ b_{C_1C_1}a_{C_1T_1} & b_{C_1C_1}a_{C_1T_1} & & & & \\ & b_{C_1C_1}a_{C_1T_1} & b_{C_1C_1}a_{C_1T_1} & & & \\ & & b_{C_1C_1}a_{C_1T_1} & b_{C_1C_1}a_{C_1T_1} & & \\ & & & b_{C_1C_1}a_{C_1T_1} & b_{C_1C_1}a_{C_1T_1} & \\ & & & & b_{C_1C_1}a_{C_1T_1} & b_{C_1C_1}a_{C_1T_1} \end{bmatrix}, \tag{32}$$

$$B_K A_{KT}^3 = B_K (A_{KT} + A_{KC} B_C A_{KC}) = \begin{pmatrix} b_{KT_1} & b_{KT_2} & b_{KT_3} & b_{KT_4} & b_{KT_5} & b_{KT_6} \end{pmatrix}$$

where $b_{KT_i} = B_K \left(a_{KT_i} + a_{KC_i} b_{C_iC_i} a_{C_iT_i} + a_{KC_{i+1}} b_{C_{i+1}C_{i+1}} a_{C_{i+1}T_i}\right), \qquad i = 1, 2, ..., 5,$ (33)

and $b_{KT_6} = B_K \left(a_{KT_6} + a_{KC_1} b_{C_1C_1} a_{C_1T_6} + a_{KC_6} b_{C_6C_6} a_{C_6T_6}\right)$

$$B_K A_{KC} = \begin{pmatrix} B_K a_{KC_1} & B_K a_{KC_2} & B_K a_{KC_3} & B_K a_{KC_4} & B_K a_{KC_5} & B_K a_{KC_6} \end{pmatrix} \tag{34}$$

Hence, the places of non-zero blocks (shown as $\otimes$) in the matrix L of the backward linkages can be seen on the following scheme:

$$\begin{bmatrix} I &&&&&&&&&&&& \\ & I &&&&&&&&&&& \\ && I &&&&&&&&&& \\ &&& I &&&&&&&&& \\ &&&& I &&&&&&&& \\ &&&&& I &&&&&&& \\ \otimes &&&&& \otimes & I &&&&&& \\ \otimes & \otimes &&&&&& I &&&&& \\ & \otimes & \otimes &&&&&& I &&&& \\ && \otimes & \otimes &&&&&& I &&& \\ &&& \otimes & \otimes &&&&&& I && \\ &&&& \otimes & \otimes &&&&&& I & \\ \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & I \end{bmatrix} \qquad (35)$$

corresponding to the scheme of backward linkages that was described in figure 2.

The structure of the income generation in this input-output central place system is given by the following diagonal matrix reflecting the income generation within towns, cities and the central city:

$$\overline{V} = \begin{bmatrix} v_{T_1T_1} &&&&&&&&&&&& \\ & v_{T_2T_2} &&&&&&&&&&& \\ && v_{T_3T_3} &&&&&&&&&& \\ &&& v_{T_4T_4} &&&&&&&&& \\ &&&& v_{T_5T_5} &&&&&&&& \\ &&&&& v_{T_6T_6} &&&&&&& \\ &&&&&& v_{C_1C_1} &&&&&& \\ &&&&&&& v_{C_2C_2} &&&&& \\ &&&&&&&& v_{C_3C_3} &&&& \\ &&&&&&&&& v_{C_4C_4} &&& \\ &&&&&&&&&& v_{C_5C_5} && \\ &&&&&&&&&&& v_{C_6C_6} & \\ &&&&&&&&&&&& v_{KK} \end{bmatrix} \qquad (36)$$

The consumption in this central place system is revealed by the following matrix whose block structure reflects the “bottom-up” spatial organization of consumption in which the income generated in the towns, cities and capital is spent also in the neighboring central places:

$$\widetilde{C} = \begin{bmatrix}
c_{T_1T_1} & c_{T_1T_2} & & & & c_{T_1T_6} & c_{T_1C_1} & & & & & c_{T_1C_6} & c_{T_1K} \\
c_{T_2T_1} & c_{T_2T_2} & c_{T_2T_3} & & & & c_{T_2C_1} & c_{T_2C_2} & & & & & c_{T_2K} \\
 & c_{T_3T_2} & c_{T_3T_3} & c_{T_3T_4} & & & & c_{T_3C_2} & c_{T_3C_3} & & & & c_{T_3K} \\
 & & c_{T_4T_3} & c_{T_4T_4} & c_{T_4T_5} & & & & c_{T_4C_3} & c_{T_4C_4} & & & c_{T_4K} \\
 & & & c_{T_5T_4} & c_{T_5T_5} & c_{T_5T_6} & & & & c_{T_5C_4} & c_{T_5C_5} & & c_{T_5K} \\
c_{T_6T_1} & & & & c_{T_6T_5} & c_{T_6T_6} & & & & & c_{T_6C_6} & c_{T_1T_1} & c_{T_6K} \\
c_{C_1T_1} & c_{C_1T_2} & & & & & c_{C_1C_1} & & & & & & c_{C_1K} \\
 & c_{C_2T_2} & c_{C_2T_3} & & & & & c_{C_2C_2} & & & & & c_{C_2K} \\
 & & c_{C_3T_3} & c_{C_3T_4} & & & & & c_{C_3C_3} & & & & c_{C_3K} \\
 & & & c_{C_4T_4} & c_{C_4T_5} & & & & & c_{C_4C_4} & & & c_{C_4K} \\
 & & & & c_{C_5T_5} & c_{C_5T_6} & & & & & c_{C_5C_5} & & c_{C_5K} \\
c_{C_6T_1} & & & & & c_{C_6T_6} & & & & & & c_{C_6C_6} & c_{C_6K} \\
c_{KT_1} & c_{KT_2} & c_{KT_3} & c_{KT_4} & c_{KT_5} & c_{KT_6} & c_{KC_1} & c_{KC_2} & c_{KC_3} & c_{KC_4} & c_{KC_5} & c_{KC_6} & c_{KK}
\end{bmatrix} \quad (37)$$

The formulae (24-37) include all blocks of the Miyazawa interrelational income multiplier (27) and can be used for its analysis.

One useful feature of this system would be the ability to trace the structure of income propagation within central place systems; for example, exploiting the notion of feedback loops, it would be possible to examine an hierarchical structure of loops based on the magnitude of the incomes flows that are generated. The structure of these loops would of course vary in the bottom-up as opposed to the top-down system. In a Löschian system, a non-regular arrangement of central places around the capital, the spatial multipliers would have different characteristics in "city rich" and "city poor" regions.

The integrated system could also be used for the analysis of the process of income distribution under various developmental conditions. For example, does the existence of the hierarchy sustain and perpetuate disparities? Under what conditions would income convergence arise? What are the critical parameters in conditioning these distributions? Obviously, the results may be very dependent on the initial conditions. Nazara *et al*. (2001) have explored a (non central place) hierarchical system to view the process of income convergence; their findings are conditioned in large part by the specification of the hierarchy. One advantage of this perspective is that it approaches the spatial system as a set of interdependent parts; the linkages between the parts are captured through an *a priori* specification of the structure that reflects an hierarchy of interactions that may be seen to be more theoretically defensible than the adoption of a binary weight matrix based on spatial contiguity. An alternative approach that exploits a Miyazawa structure attempted to build an hierarchy of complication of interactions in a system of twelve regions within the Chicago metropolitan area (Hewings *et al*., 2003). The complication builds up from interindustry flows, income flows and consumption flows in a way that reflects the circulation of goods, factors (labor inputs), income to households and finally consumption expenditures – each with a different spatial structure.

9. Conclusions

This paper has attempted to unify two important paradigms in regional science – classical input-output analysis with Miyazawa's modifications and the classical Christaller-Lösch central place theory. While retention of the basic consumption principles of central place theory can be retained, there are many alternatives for the specification of the production relationships. Two options, a top-down and bottom-up, were presented and interpreted through an LDU decomposition. However, there are many more possibilities that could be explored, particularly those associated with the daily mobility (journey-to-work) of labor and assumptions about the location of expenditures from income (journey-to-shop). As revealed by Hewings *et al*. (2003), when shopping expenditures are made at other than the location of the residence of the income earner, the structure of income interdependence (Miyazawa's interrelational income multiplier) becomes much more complex.

The considerations presented here then may be considered as the initial steps towards a general approach to the integration of spatial and sectoral economic structure. The challenge of incorporating a temporal dimension presents yet another opportunity for exploring the analysis of organization of production in geographical space and the degree to which this structure evolves with new innovations, new products and institutional changes (such as free trade agreements).

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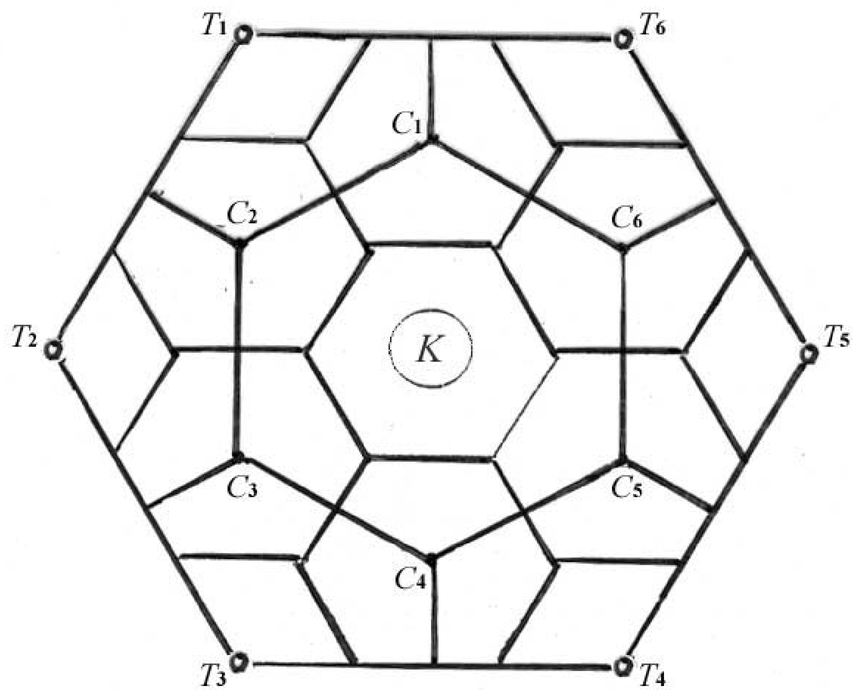

Figure 1. Beckmann-McPherson three tier Central Place system

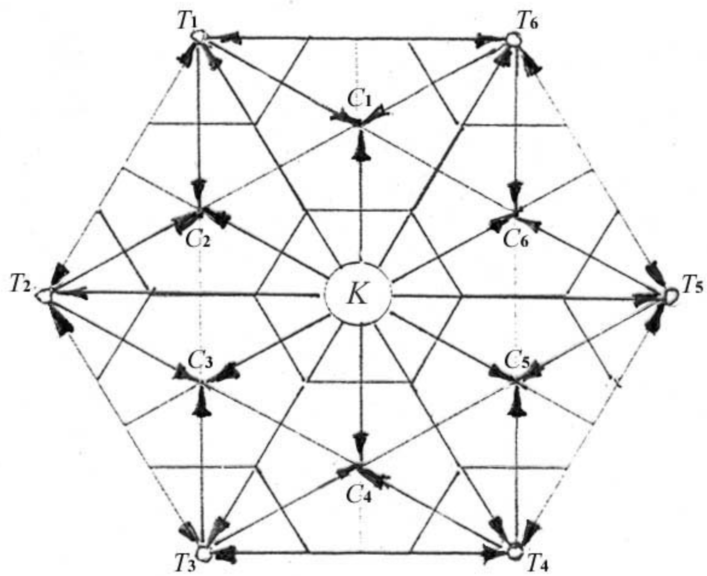

Figure 2: “Top-down” structure of transaction flows in input-output three tier Beckmann-McPherson central place system