STRUCTURAL CHANGE DECOMPOSITION THROUGH GLOBAL SENSITIVITY ANALYSIS OF INPUT-OUTPUT MODELS

by

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Structural Change Decomposition Through Global Sensitivity Analysis of Input-Output Models*

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Abstract: Sensitivity analysis has become an important tool to test the robustness of estimated economic models. In this paper we propose the use of simulation-based sensitivity analysis to identify the fundamental structure of the economy. To show the possibilities of this technique, we provide empirical evidence on the path of structural change occurring in the Chicago economy by running simulation in projected input-output tables over the period 1975-2010.

Keywords: Input-Output Models, Sensitivity Analysis, Importance Matrix
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1. Introduction

One of the main issues in input-output models is the fact that they are substantially deterministic in the technological parameters. However, as pointed out in several studies (Bullard and Sebald, 1977; Jackson, 1986; Israilevich *et al*., 1996), there are at least four sources of uncertainty in input-output models:

a) if the input-output model is survey-based, then there could be a classical sampling error;

b) in the case of large surveys, an error in the inference design can arise;

c) real technical coefficients are not constant over the time, and in an age of structural change due to technological development, this error can be relevant;

d) errors in compiling the large data base can affect the quality of the final table.

These issues have been addressed by scholars since the beginning of the widespread application of input-output models. For example, Quandt (1958 and 1959), ran a primitive sensitivity analysis by disturbing the error distributions and then observing the change in output. Since then, three different paradigms, attempting to analyze the stochastic behavior of technical coefficients in an input-output framework, have been proposed. First, following the work of Simonovitz (1975), Lahiri (1983) and Lahiri and Satchel (1985) have provided some conditions for the over- and under-estimation of the Leontief matrix (and of the output) by assuming biproportional stochastic independence in the elements. The second approach relies on the theoretical work by Sherman and Morrison (1950) who analyzed the effect on the inverse of a change in an element of the original matrix. In this context, Sonis and Hewings (1992) developed the well-known theory of the fields of influence and of the deterministic sensitivity analysis. On the other hand, West (1986) extended the results of Sherman and Morrison (1950) by considering the case of a stochastic Leontief inverse. The third paradigm is hybrid, in the sense that it considers the possibility of updating the input-output tables through econometric models (see Kraybill, 1991; Conway, 1990; Treyz and Stevens, 1985; Treyz, 1993). In this context, Israilevich *et al.* (1997) provide an interesting approach to structural change forecasting by considering an innovative framework based on the general assumptions of the computable general equilibrium models.

In a recent paper (which is closest to the present one), Rey *et al.* (2003) examine the role of uncertainty in integrated econometric+input-output regional models. By running a series of Monte Carlo simulations they analyzed three sources of uncertainty:

1) econometric model parametric uncertainty;

2) econometric disturbance term uncertainty;

3) input-output coefficient uncertainty.

The aim of their study was the identification what source of uncertainty is the most important, but the results suggest that the answer is not unequivocal and does depend on model specification as well as on contingent characteristics.

The present paper revisits a partially abandoned approach in the economic literature, that is, the simulation analysis in the context of input-output models. In particular, since the work by Bullard and Sebald (1988) appeared, few papers have provided further insights. Yet, as Jackson (1986) noted, there are significant reasons to consider a probabilistic approach in an input-output context. In particular, he writes that: *"the set of all like coefficients for an industry for each of m regions defines m subpopulations, and an associated probability of realizing a particular coefficient within the total population or within each subpopulation"*. In addition, as long as we consider a probabilistic approach, the term "error" is no longer appropriate because the probability distribution associated with each technical coefficient is meant to provide the complete range of possible realizations.

The aim of this paper is to present an innovative procedure in order to run quantitative sensitivity analysis in the context of input-output models. In particular, we propose the computation of an *Importance Matrix* defined as the ordered set of the indices of sensitivity associated with each linkage of the Leontief matrix. This new concept provides a quantitative measure of the relative importance for the economy of each element and of each sector. The analysis will be explored with an empirical application to the Chicago economy.

The paper is organized as follows. In section 2 we review the possibilities of sensitivity analysis in economic modeling; in the third section the simulation-based global sensitivity analysis is presented. Section 4 contains an application of the proposed methodology, while concluding remarks comprise the remaining section.

2. On the Use of Sensitivity Analysis in Economic Models

As defined by Baird (1989), sensitivity analysis in the investigation of potential changes in parameter values and assumptions of any economic models and their impacts on conclusions to be drawn from the model. Fiacco (1983) states that "*a sensitivity and stability analysis should be an integral part on any solution methodology. The status of a solution cannot be understood without such information*". These considerations allow scholars to consider sensitivity analysis as a fundamental ingredient in the making of scientific inquiry through models (Saltelli *et al.*, 2002; Saltelli and Tarantola, 2003).

From an economic theoretical viewpoint, the effects of a changing environment on optimal economic decisions have been subject of much research. The leading method of sensitivity analysis is based on applying the implicit function theorem to a system of equations representing optimality.[1] However, in recent years, a number of studies in the various areas of applied economics have also appeared on several journals.[2]

Even though sensitivity analysis has been conceived to test the robustness of estimated models, Pannell (1997) suggests a wide range of uses to which this technique is put. In table 1, the uses are grouped into four main categories: decision making or development of recommendations for decision making, communication, increased understanding of quantification of the system, and model development.

<<insert table 1 here>>

For the purpose of the present paper, it is evident that the third group of uses is of particular interest for the identification of the key structure of an economy. In particular, following a previous article by Percoco *et al.* (2004), exogenous shocks to selected variables in the context of input-output models are imposed in order to observe the relative change in the production of the economy.[3] Thus, sensitivity analysis is here meant to improve the understanding of the

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[1] On this point see the classical reading of Samuelson (1947) and Silberberg (1974). Chavas (2001) presents an interesting extension of local sensitivity analysis to take into account wider changes in economic environment and related impact on optimal choice problems.
[2] See, among others, Canova (1995), Dungon and Wilson (1991), Eschenbach and Gimpel (1990), Eschenbach and McKeague (1989), Harrison and Vinod (1992), Nordblom *et al.* (1994).
[3] A very close paper this one is Rey *et al.* (2003), in which the author study the impact of three sources of uncertainty (parameters, disturbances and input-output coefficients) in the context of an integrated econometric+input-output regional model.

relationships between inputs and outputs, i.e., we use the proper tools of uncertainty analysis to assess the relative importance of model (economy) parameters (relationships).

It should be stated that methodology we use is quite close to the *fields of influence approach* as proposed by Sonis and Hewings (1992), where a deterministic change in the elements of the matrix of inter-industry linkages is imposed and then the ramification of induced changes in the Leontief inverse is observed. The main difference with our approach centers on the fact that we consider a random change through the assumption of a probability function and then, by running a simple Monte Carlo simulation, we are allowed to compute indices of relative importance and thus to identify the most important sectors (Bullard and Sebald, 1988). The Sonis and Hewings (1992) approach, in contrast, explores the impact of an arbitrary change in the coefficients (a single coefficient, two or more coefficients, a row or column) on the output vector; thus, there is no appeal to a probability distribution of changes. The present approach takes the idea of a field of influence of change but presents it in a probabilistic framework.

Furthermore, there are a number of complementary approaches to the issue of coefficient change that need to be highlighted. The issue of sensitivity analysis occurs in the identification of appropriate procedures for updating input-output matrices. The popular biproportional methodology has recently been revisited and expanded by de Mesnard (1990, 1997, 2000) while an empirical exploration by Okuyama *et al*. (2002) compared a more general equilibrium updating procedure with a simple biproportional adjustment. The present methodology may provide a feasible alternative in that it is rooted in the production function approach to handling coefficient change through the specification of de facto bounded uncertainty in the estimation procedure and its impact on system-wide production.

3. The Basic Framework

Let us consider the classical input-output model in the form:

$$\mathbf{X} = (\mathbf{I}\text{-}\mathbf{A})^{-1}\mathbf{F} \tag{1}$$

where $\mathbf{X}$ is an $n \times 1$ vector of the total output of the n sectors in the economy, $\mathbf{F}$ is the $n \times 1$ vector of the final demand and $(\mathbf{I}\text{-}\mathbf{A})^{-1}$ is the Leontief inverse matrix. In general, we can write (1) as:

$$\mathbf{X} = f[(\mathbf{I}\text{-}\mathbf{A})^{-1}, \mathbf{F}] \tag{2}$$

The aim of this paper is to assess the volatility in the elements of $(\mathbf{I}-\mathbf{A})^{-1}$ and how it explains the variance V of the total output. Let us suppose, for the moment, that all these coefficients are affected by uncertainty and that the elements of **F** are known, so that, for the purpose of Sensitivity Analysis (SA) we can write:

$$\mathbf{X} = f(a_{11},\ldots,a_{ij},\ldots,a_{nn}) \tag{3}$$

where a_{ij} is the generic element of matrix **A.** By using a simple ANOVA decomposition, we can write the variance of the output as:

$$V(x_i) - E\left[V\left(x_i \middle| a_{ij}\right)\right] = V\left[E\left(x_i \middle| a_{ij}\right)\right] \tag{4}$$

Equation (4) is a good measure of the sensitivity of the i^{th} sector output with respect to the inter-industry linkage. In particular, the importance of a_{ij} relates to how well a_{ij} drives changes in x_i, that is how well $E\left[x_i \middle| a_{ij}\right]$ mimics x_i. If the total variation in x_i is matched by the variability in $E\left[x_i \middle| a_{ij}\right]$ as a_{ij} varies, this implies that a_{ij} could be a very important inter-industry linkage; that variation is measured by the term $V\left[E\left(x_i \middle| a_{ij}\right)\right]$. The term $E\left[V\left(x_i \middle| a_{ij}\right)\right]$ can be described as a prediction error, measuring the remaining variability in sector output. If we divide equation (4) by the unconditional variance, we obtain the *index of sensitivity*:

$$S_{ij} = \frac{V\left[E\left(x_i \middle| a_{ij}\right)\right]}{V(x_i)} \tag{5}$$

that is scaled in [0,1]. Note that the complete set of sensitivity indices in equation (5) is an $n \times n$ matrix and it provides a more specific formulation of the importance measures provided by Bullard and Sebald (1977). In what follows, this set will be called the *Importance Matrix.* We assume that the columns are the reacting sectors, i.e. the sectors $i = 1,\ldots,n$ affected by a change in the value of a_{ij}.[4] In the rows, there are the activating sector, i.e. the sectors $j = 1,\ldots,n$ whose technological change is meant to generate volatility on the reacting sectors. In a regional input-output system or a system or a very open national economy, the change could be generated by changes in regional supply; for example, Hewings et al., (1998) noted the existence of a hollowing out phenomenon in the Chicago economy whereby interregional trade replaced

[4] A similar idea was proposed by Van der Linden *et al*. (2000).

intraregional sales and purchases. Globalization tendencies have made such phenomena more common but usually referred to as a fragmentation process (see Jones and Kierzkowski, 2001).

3.1 Second Order and Total Sensitivity Indices

In equation (3), we have implicitly assumed that the unique factors affected by uncertainty were the elements of the matrix **A**. In this section, we will remove this assumption and will consider also the uncertainty in the elements of the vector **F**, so that we are interested in the computation of the importance of the interaction between **A** and **F**. In this case, the overall effect of the elements of the final demand (which can be considered as a trial set) on the variance of the output can be expressed as:

$$V\left[E\left(x_i \middle| a_{ij}\right)\right] + V\left[E\left(x_i \middle| a_{ij}, f_i\right)\right] = V(x_i) + V\left[E\left(x_i \middle| f_i\right)\right] \tag{6}$$

In equation (6), $V\left[E\left(x_i \middle| a_{ij}\right)\right]$ is the first order effect of technological change as described in the previous section, $V\left[E\left(x_i \middle| f_i\right)\right]$ is the variance of the output due to a change in the final demand, while $V\left[E\left(x_i \middle| a_{ij}, f_i\right)\right]$ is the interaction term between the elements of matrix **A** and the ones of vector **F**. If $V\left[E\left(x_i \middle| a_{ij}\right)\right] \cong V(x_i)$, then **F** is non-influential, and all factors in **F** can be fixed in a subsequent analysis of the model.

At this point we can introduce the total index of sensitivity for the generic i^{th}, j^{th} linkage defined as:

$$S_{ij}^{T} = \frac{V\left[E\left(x_i \middle| a_{ij}\right)\right]}{V(x_i)} + \frac{V\left[E\left(x_i \middle| f_i\right)\right]}{V(x_i)} + \frac{V\left[E\left(x_i \middle| a_{ij}, f_i\right)\right]}{V(x_i)} \tag{7}$$

where the first terms of the right-hand side are the first order sensitivity indices for the interindustry and final demand changes respectively, while the last term is meant to measure the importance of the interaction of the two variables in explaining the volatility of the output of sector *i*.

4. The Decomposition of Structural Change of the Chicago Economy

As stated above, the aim of this paper is to provide a new look at the evidence on the hollowing out process occurring in the urban economy of Chicago by using a variance decomposition approach for the identification of the most important sectors. The main assumption is that the temporal pattern of the relative importance of a given sector is useful to highlight the sectors driving the structural change.

In order to provide a simple example of the procedure described in section 3, we use the Chicago input-output table for the year 2000.[5] This table contains just 9 sectors: Resources (RES), Construction (CONST), Non-Durable Goods (NDG), Durable Goods (DG), Transportation (TRANS), Trade (TRADE), Financial Services and Real Estate (FIRE), Services (SER) and Government (GOV).

The first step is to assign a distribution function to the technical coefficients. According to Bullard and Sebald (1988), and in contrast to Jackson (1987), we will consider a lognormal distribution with a 99.7% confidence interval and run it for 1000 runs. In addition, we use the explicit formulation of the technical coefficients so that we do not need to apply an exclusion procedure for the sample in which the column sum differs from 1. It should also be noted that by using an assumption for the probability function and for the confidence interval, the structure of the simulated matrices is consistent with the theory of entropy in input-output systems (see Sonis *et al.*, 2000).

To explain this point, let us consider the general matrix filling problem of finding matrix element b_{ij} consistent with row target totals $a_{\bullet j}$ and column target totals $a_{i\bullet}$. This problem has the trivial solution, namely, to allocate values equiproportionally across columns and rows:

$$b_{ij} = \frac{a_{i\bullet}}{I}$$

where $I = \sum_j a_{\bullet j} = \sum_i a_{i\bullet}$ is the intensity of the matrix. Carrying out a simulation of n runs by using the previous formula for b_{ij} changes the procedure to the problem of finding n matrices subject to row and column constraints.

[5] For further details on the construction of the tables, see Israilevich *et al.*(1997).

As demonstrated in McDougall (1999), proportional allocation is a maximum entropy model, thus the choice of using an explicit formulation for b_{ij} with equal distribution across all coefficients will result in a series of simulated matrices all showing maximum entropy. The implication of this choice is that the output of sector i depends not only on the transactions (coefficients) a_{ij} where j=1,…,n, but also, in a non-linear, thus interacting, way from all the n^2 linkages. For the sake of simplicity, in this paper we will not consider all these indices of sector interactions.

In table 2, we present the *First Order Importance Matrix* **S**. The columns are the reacting sectors, i.e. the sectors $i=1,...,n$ affected by a change in the generic element of **A**, a_{ij}. In the rows, there are the activating sector, i.e. the sectors $j=1,...,n$ whose technological change is meant to generate a change on the sectors $i=1,...,n$. This, in turn, means that the generic element of the matrix measures the effect on the output of the sector i of a change of the technical coefficient a_{ij}.

As expected, for the Resources and Construction sectors, the variables SER, TRADE and DG present the highest values (figures 1a and 1b). In addition, the transportation sector (TRANS) has a great importance for the durable goods industry (figure 1e), while the public expenditure does not show relevant values (figures 1a-1i), only for the RES sector does it show a Pearson Coefficient different from zero.

<<insert figures 1a-1i here>>

In the context of the importance matrix, we can compute the following marginal index:

$$S_i = \sum_j S_{ij} \tag{8}$$

that measures the absolute importance of the sector j for the economy as a whole and will be called an *index of absolute importance*[6]. Note that $S_i : [0,1] \rightarrow [0,n]$. Table 3 reveals that the durable goods industry and services present the highest values, implying a strong dependence of the Chicago economy on these two sectors.

<<Insert table 3 about here>>

In addition, we can compute a synthetic index of reaction by considering:

[6] Recall that the index of sensitivity S_{ij} measures the relative importance of the inter-linkage a_{ij} for sector i, thus the column sum of these indices can be thought as the overall importance of sector i for the economy as a whole.

$$S_j = \sum_i S_{ij} \tag{9}$$

This index (an *index of absolute sensitivity;*[7] $S_j : [0,1] \rightarrow [0,n]$) provides a measure of the aggregate volatility of the *n* sectors. Table 4 shows the results for the Chicago economy. Even if the difference among the indices of absolute sensitivity is quite narrow (the standard error is 0.118), the durable goods sector again and RES present the highest values, implying a higher sensitivity of these sector to structural changes in the economy. Of course, one would expect the differences to increase with greater levels of sectoral disaggregation. Notice that the two synthetic indices proposed in (8) and (9) are very similar to the Hirschman-Rasmussen indices of backward and forward linkages. In particular, the power of dispersion for the backward linkages for sector *j* is described by a weighted average of column multiplier, as the index of absolute importance is defined as the column sum of importance indices. Similarly, the index of the sensitivity of dispersion for forward linkages for sector *i* is defined as the weighted average of row multiplier, while the index of absolute sensitivity is the row sum of importance measures. Thus, both classes of indices are meant to measure the relative importance or sensitivity of a given sector, with the main difference that the Hirschman-Rasmussen indices are conceived as mean values, while the indices we propose in this paper are simple aggregations of importance measures.

<<Insert table 4 about here>>

We now use the total sensitivity index to account for the interaction between **F** and **A**. In particular, let us recall the formula:

$$S_{ij}^{T} = S_{ij}^{\mathbf{A}} + S_{ij}^{\mathbf{F}} + S_{ij}^{\mathbf{AF}} \tag{10}$$

where $S_{ij}^{\mathbf{A}}$ and $S_{ij}^{\mathbf{F}}$ are the first order sensitivity indices of matrix **A** and vector **F** respectively, $S_{ij}^{\mathbf{AF}}$ is the importance measure for the interaction between them. With these indicators in mind we can identify whether the importance of a given linkage is likely to be driven by technological change, by change in the final demand or both.[8] In this way, the methodology may provide a

[7] Similarly to the case of index in (8), the row sum of indices of sensitivity can be interpreted as the sensitivity of sector *j* to changes occurring over all the *n* sectors of the economy.
[8] Notice that in the empirical application that follows we will neglect the term $\mathbf{S}^{\mathrm{F}}$ because we are mainly interested in assessing the importance of final demand in generating structural changes of the economy.

complement to the more familiar structural decomposition approaches to the measurement of structural change (see Sonis *et al*., 1996; Dietzenbacher and Hoekstra, 2002). In these approaches, various methods have been employed to separate out the importance of coefficient change, changes in final demand and interaction effects in accounting for changes in output. The present approach allows us to point out the sources of structural change over the time by observing the pure technological change, as expressed by changes in the Leontief inverse and the interaction between conjunctural changes, as expressed by movements in vector **F**, and variations in matrix **A**. Tables 5 and 6 present the total and interaction terms matrices and results suggest at least two comments. First, with the sole exception of services, all the sectors present higher indices of sensitivity in the case of interaction of structural and conjuncture changes. In table 6 it is worth noting that durable goods and the service industries are the most important for the Chicago economy.

<<insert tables 5 and 6 here>>

In order to decompose the structural change occurring in the Chicago economy over the period 1975-2011, we ran the same experiment for each year. Using the same dataset, Hewings *et al.* (1998) find out a hollowing-out process, with intrametropolitan dependence replaced by dependence on sources of supply and demand outside the region. Furthermore, the analysis reveals a complex internal transformation, as dependence on locally sourced manufacturing inputs is replaced by dependence on local service activities.

By considering the Chicago backward and forward linkage hierarchies, as shown in Figure 2a, Hewings *et al*. (1998) found that:

a) in general, all the internal multipliers declined: the Chicago region has thus exchanged its prior internal dependence on intermediation for external dependence reflecting a congruence of trends that involve the interplay of outsourcing, changes in ownership patterns, and increased intrasector specialization in the face of a general trend for regions to become more similar in terms of their macroeconomic structure;

b) while all sectors decreased their levels of internal dependence, there were some important differences, reflected in large part in the dominance or lack of it of intrasectoral sales and purchases;

c) while there seems to be overwhelming evidence that regional specialization is decreasing, the indicators that are used in the analysis are based on vector realizations of the distribution of output or employment

By using the simulation-based procedure presented in this paper, we observe (figure 2b) an anticipation of the aforementioned structural change, as depicted by the temporal patterns of the indices of absolute importance and sensitivity calculated over the Total Importance Matrix.

<<insert figure 2 about here>>

It would seem that the new index and the Hirschman-Rasmussen indices present the same rankings, but the main advantage is that the total index S_{ij}^{T} can be easily decomposed into several terms, helping identify the sources of change in the economy. On this point, table 7 shows the decomposition of the variance of the proposed indices of sensitivity and importance, calculated over the time period 1975-2010.

5. Concluding remarks

In this paper we have provided a new and promising methodology to assess the effect on the output of the uncertainty in the technical coefficients of an input-output model. After describing the simulation design and the computational procedure, we have proposed four innovative importance measures: the index of sensitivity , the Importance Matrix, the index of absolute importance and the index of absolute sensitivity. By using a decomposition of the changes occurring in these measures, it is possible to decompose the structure of the effect on the sectors of the economy of a structural change .

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Appendix

The computational procedure[9]

Let us consider the following formulation of the input-output model:

$$x = f\left(a_{1i},...,a_{ij},...,a_{nn}\right) \tag{A1}$$

If the generic element a_{ij} is fixed to a generic value $\tilde{a}_{ij}$, then the variance of sector x_i can be written as:

$$\begin{aligned} V\left(x\middle|a_{ij}=\tilde{a}_{ij}\right) &= \int...\int\left[f\left(a_{1i},...,\tilde{a}_{ij},,,a_{nn}\right)-E\left(x_i\middle|a_{ij}=\tilde{a}_{ij}\right)\right]^2\prod_{i\neq j}p_i(a_i)da_i = \\ &= \int...\int f\left(a_{1i},...,a_{ij},...,a_{nn}\right)\prod_{i\neq j}p_i(a_i)da_i - \left[f\left(a_{1i},...,\tilde{a}_{ij},,,a_{nn}\right)-E\left(x_i\middle|a_{ij}=\tilde{a}_{ij}\right)\right]^2 \end{aligned} \tag{A2}$$

where *p(a)* is the probability distribution of *a*. For the purpose of sensitivity analysis, one is interested in eliminating the dependence upon the value of $\tilde{a}_{ij}$ by integrating $V\left(x_i\middle|a_{ij}=\tilde{a}_{ij}\right)$ over the probability density function of $\tilde{a}_{ij}$, obtaining:

$$E\left[V\left(x_i\middle|a_{ij}\right)\right] = \int...\int\left[f(\bullet)\right]^2\prod_i p_i(a_i)da_i - \int E\left(x_i\middle|a_{ij}=\tilde{a}_{ij}\right)p_j(\tilde{a}_{ij})d\tilde{a}_{ij} \tag{A3}$$

Notice that we have dropped the dependence $\tilde{a}_{ij}$ from the left-hand side, as it disappears due to the integration. Let us define the variance of x_i as:

$$V\left(x_i\right) = \int...\int\left[f(\bullet)\right]^2\prod_i p_i(a_i)da_i - \left[E\left(x_i\right)\right]^2 \tag{A4}$$

By subtracting Eq (A3) from Eq. (A4) we obtain:

$$V\left(x_i\right) - E\left[V\left(x_i\middle|a_{ij}\right)\right] = \int\left[E\left(x_i\middle|a_{ij}=\tilde{a}_{ij}\right)\right]^2 p_j(\tilde{a}_{ij})d\tilde{a}_{ij} - \left[E\left(x_i\right)\right]^2 \tag{A5}$$

[9] The present Appendix heavily relies on Saltelli (2002).

The problem is that (A5) is computationally impractical. In a Monte Carlo framework, it implies a double loop: the inner to compute $\left[E(x_i)\right]^2$ and outer to compute the integral. For this reason, we can rewrite the (A5) as:

$$V(x_i) - E\left[V(x_i | a_{ij}\right] = \int ... \int f(\cdot) f(a^{'}_{i1},...,a^{'}_{ij},...,a^{'}_{in}) \prod_i p_i(a_i) da_i \prod_i p_i(a'_i) da'_i \\ - \left[E(x_i)\right]^2 \tag{A6}$$

The expedient of using the additional integration variable primed, allows us to realize that the integral in the previous equation is the expectation value of the function *f* on a set of (2*n* – 1) technical coefficients. Now, this integral can be computed using a single Monte Carlo loop.

Let us generate two sample matices $\mathbf{M}_1$ and $\mathbf{M}_2$ for the inter-industry coefficients:

$$\mathbf{M}_1 = \begin{pmatrix} a_{111} & ... & a_{1ij} & ... & a_{1nn} \\ a_{211} & ... & a_{2ij} & ... & a_{2aa} \\ ... & ... & ... & ... & ... \\ a_{k11} & ... & a_{kij} & ... & a_{knn} \end{pmatrix}$$

$$\mathbf{M}_2 = \begin{pmatrix} a'_{111} & ... & a'_{1ij} & ... & a'_{1nn} \\ a'_{211} & ... & a'_{2ij} & ... & a'_{2aa} \\ ... & ... & ... & ... & ... \\ a'_{k11} & ... & a'_{kij} & ... & a'_{knn} \end{pmatrix}$$

where *k* is the sample size used for the Monte Carlo estimate. In order to estimate the sensitivity measure for the generic element a_{ij}, i.e.

$$S_{ij} = \frac{V\left[E\left(x_i | a_{ij}\right)\right]}{V(x_i)} = \frac{U_{ij} - \left[E(y_i)\right]^2}{V\left(x_i\right)}$$

$$U_{ij} = \int \left[E\left(x_i | a_{ij} = \tilde{a}_{ij}\right)\right]^2 p_j(\tilde{a}_{ij}) d\tilde{a}_{ij}$$

we need an estimate for both $E[x_i]$ and U_{ij}. The former can be obtained from the values of x computed on the sample in $\mathbf{M}_1$ of $\mathbf{M}_2$. U_{ij} can be obtained from values of x_i on the following matrix:

$$\mathbf{N}_{ij} = \begin{pmatrix} a'_{111} & \dots & a_{1ij} & \dots & a'_{1nn} \\ a'_{211} & \dots & a_{2ij} & \dots & a'_{2aa} \\ \dots & \dots & \dots & \dots & \dots \\ a'_{k11} & \dots & a_{kij} & \dots & a'_{knn} \end{pmatrix}$$

i.e. by

$$\tilde{U}_{ij} = \frac{1}{k-1}\sum_{r=1}^{k} f(\cdot)\, f(a'_{r1},\dots,a'_{rn}) \qquad \text{(A7)}$$

If one thinks of matrix $\mathbf{M}_1$ as the “sample matrix,” and of $\mathbf{M}_2$ as the “re-sample” matrix, the $\tilde{U}_{ij}$ is obtained from products of values *f* computed from the sample matrix times values of *f* computed from **N**, i.e. a matrix where all factors except a_{ij} are re-sampled. In this way, the computational cost associated with a full set of first order indices S_{ij} is *k(n+1)*. One set of *k* evaluations of *f* are needed for the second term of the product in (A7). This means that the total cost of this procedure is *2k(n+1)*.

Table 1 *Uses of Sensitivity Analysis*

Decision making or development of recommendations for decision makers	- Testing the robustness of an optimal solution; - Identifying critical values, thresholds or break-even values where the optimal strategy changes; - Identifying sub-optimal solutions; - Developing flexible recommendations which depend on circumstances; - Comparing the values of simple and complex strategies.
Communication	- Making recommendations more credible, understandable, compelling, or persuasive; - Allowing decision makers to select assumptions; - Conveying lack of commitment to any single strategy.
Increased understanding or quantification of the system	- Estimating relationships between input and output variables; - Understanding relationships between input and output variables; - Developing hypotheses for testing.
Model development	- Testing the model for validity or accuracy; - Searching for errors in the model; - Simplifying the model; - Calibrating the model; - Coping with poor or missing data; - Prioritizing acquisition of information.

Source: Pannell (1997)

Table 2 *First Order Importance Matrix for the Chicago Economy (2000)*

	RES	CONST	NDG	DG	TRANS	TRADE	FIRE	SER	GOV
RES	0,010	0,085	0,009	0,037	0,102	0,074	0,025	0,079	0,063
CONST	0,231	0,126	0,206	0,253	0,194	0,284	0,158	0,223	0,119
NDG	0,287	0,224	0,437	0,273	0,322	0,335	0,336	0,288	0,261
DG	0,410	0,390	0,450	0,438	0,505	0,470	0,466	0,440	0,454
TRANS	0,276	0,296	0,161	0,370	0,294	0,272	0,127	0,263	0,291
TRADE	0,428	0,414	0,402	0,410	0,236	0,387	0,413	0,328	0,466
FIRE	0,165	0,085	0,054	0,157	0,114	0,045	0,002	0,034	0,088
SER	0,610	0,598	0,661	0,624	0,667	0,562	0,661	0,757	0,653
GOV	0,104	0,067	0,009	0,067	0,037	0,020	0,044	0,004	0,033

Table 3 *Index of Absolute Importance*

Sector	**Index**
SER	5,793
DG	4,023
TRADE	3,484
NDG	2,763
TRANS	2,350
CONST	1,794
FIRE	0,744
RES	0,484
GOV	0,385

Table 4 *Index of Absolute Sensitivity*

Sector	Index
DG	2,629
RES	2,521
TRANS	2,471
TRADE	2,449
GOV	2,428
SER	2,416
NDG	2,389
CONST	2,285
FIRE	2,232

Table 5 *Importance Matrix for the interaction terms*

	RES	CONST	NDG	DG	TRANS	TRADE	FIRE	SER	GOV
RES	0.624	0.610	0.667	0.562	0.653	0.757	0.661	0.598	0.661
CONST	0.438	0.410	0.505	0.470	0.454	0.440	0.450	0.390	0.466
NDG	0.410	0.428	0.236	0.387	0.466	0.328	0.402	0.414	0.413
DG	0.273	0.287	0.322	0.335	0.261	0.288	0.437	0.224	0.336
TRANS	0.370	0.276	0.294	0.272	0.291	0.263	0.161	0.296	0.127
TRADE	0.253	0.231	0.194	0.284	0.119	0.223	0.206	0.126	0.158
FIRE	0.157	0.165	0.114	0.045	0.088	0.034	0.054	0.085	0.002
SER	0.037	0.010	0.102	0.074	0.063	0.079	0.009	0.085	0.025
GOV	0.067	0.104	0.037	0.020	0.033	0.004	0.009	0.067	0.044

Table 6 *Total Importance Matrix*

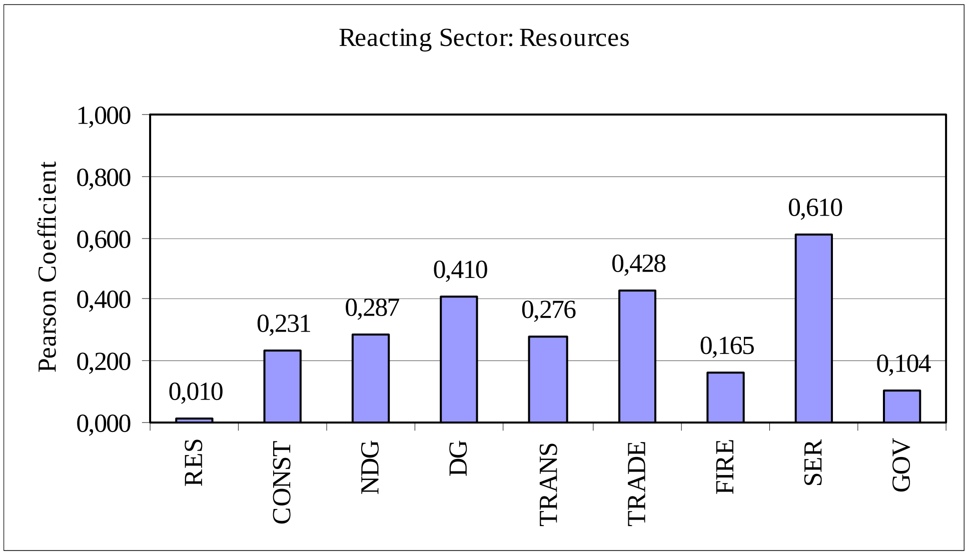
	RES	CONST	NDG	DG	TRANS	TRADE	FIRE	SER	GOV
RES	0.634	0.695	0.676	0.599	0.755	0.831	0.686	0.677	0.724
CONST	0.669	0.536	0.711	0.723	0.648	0.724	0.608	0.613	0.585
NDG	0.697	0.652	0.673	0.660	0.788	0.663	0.738	0.702	0.674
DG	0.683	0.677	0.772	0.773	0.766	0.758	0.903	0.664	0.790
TRANS	0.646	0.572	0.455	0.642	0.585	0.535	0.288	0.559	0.418
TRADE	0.681	0.645	0.596	0.694	0.355	0.610	0.619	0.454	0.624
FIRE	0.322	0.250	0.168	0.202	0.202	0.079	0.056	0.119	0.090
SER	0.647	0.608	0.763	0.698	0.730	0.641	0.670	0.842	0.678
GOV	0.171	0.171	0.046	0.087	0.070	0.024	0.053	0.071	0.077

Table 7 *ANOVA decomposition of the temporal pattern of aggregate indices*

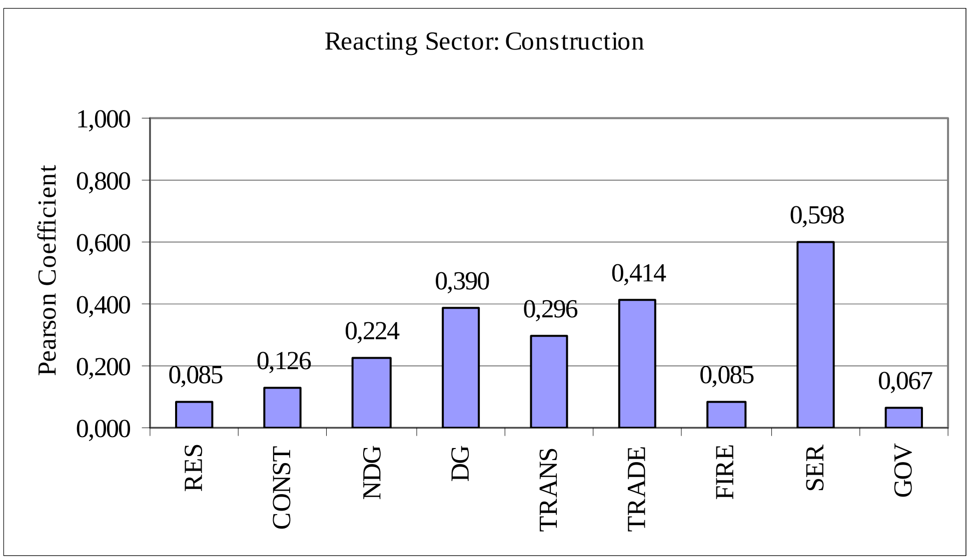
	Index of absolute importance		Index of absolute sensitivity	
	S^{A}	S^{AF}	S^{A}	S^{AF}
RES	0.17	0.83	0.24	0.76
CONST	0.41	0.59	0.58	0.42
NDG	0.76	0.24	0.32	0.68
DG	0.66	0.34	0.57	0.43
TRANS	0.19	0.81	0.46	0.54
TRADE	0.11	0.89	0.09	0.91
FIRE	0.55	0.45	0.59	0.41
SER	0.89	0.11	0.07	0.93
GOV	0.21	0.79	0.28	0.72

Figure 1 *Reacting sectors*

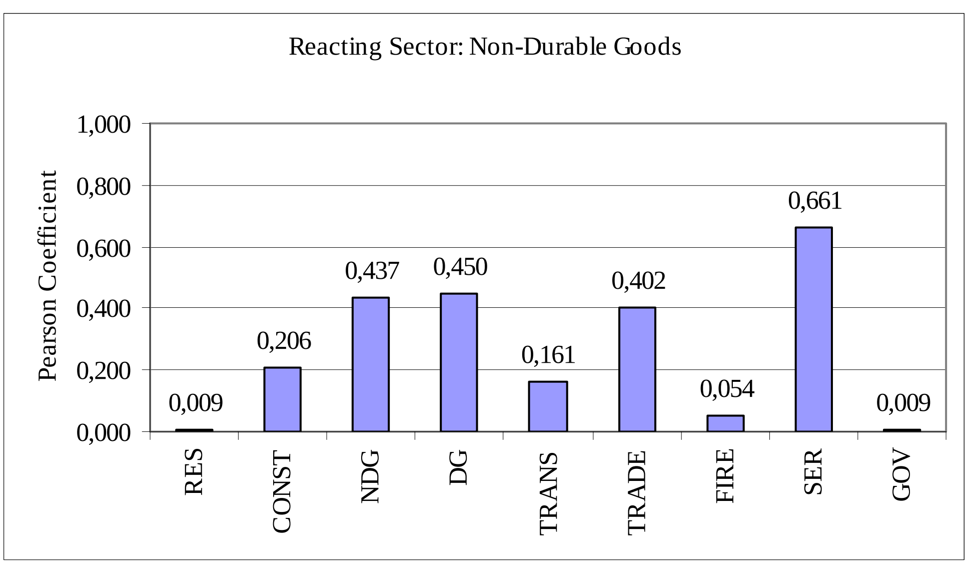
a)



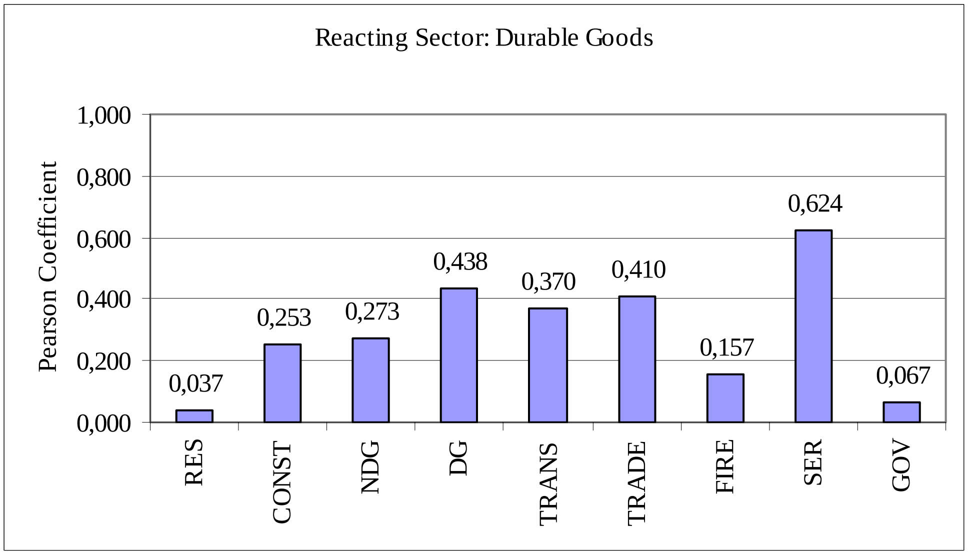
b)



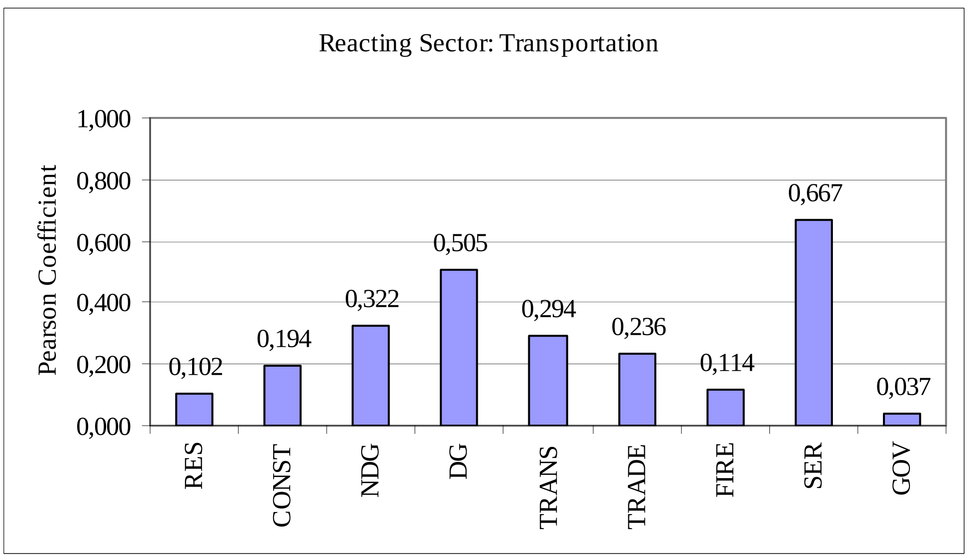
c)



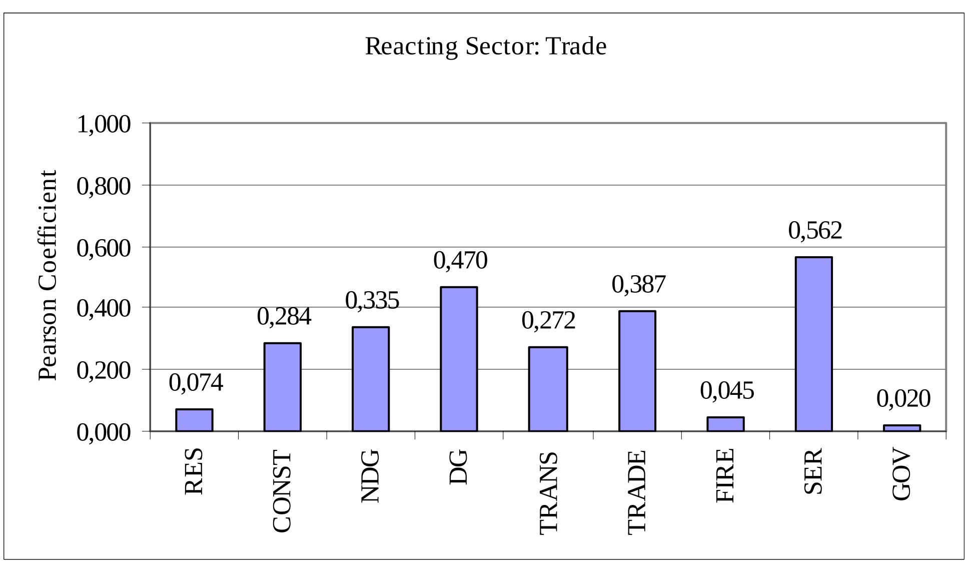
d)


Reacting Sector: Durable Goods
Pearson Coefficient
1,000
0,800
0,600
0,400
0,200
0,000
0,037
0,253
0,273
0,438
0,370
0,410
0,157
0,624
0,067
RES
CONST
NDG
DG
TRANS
TRADE
FIRE
SER
GOV

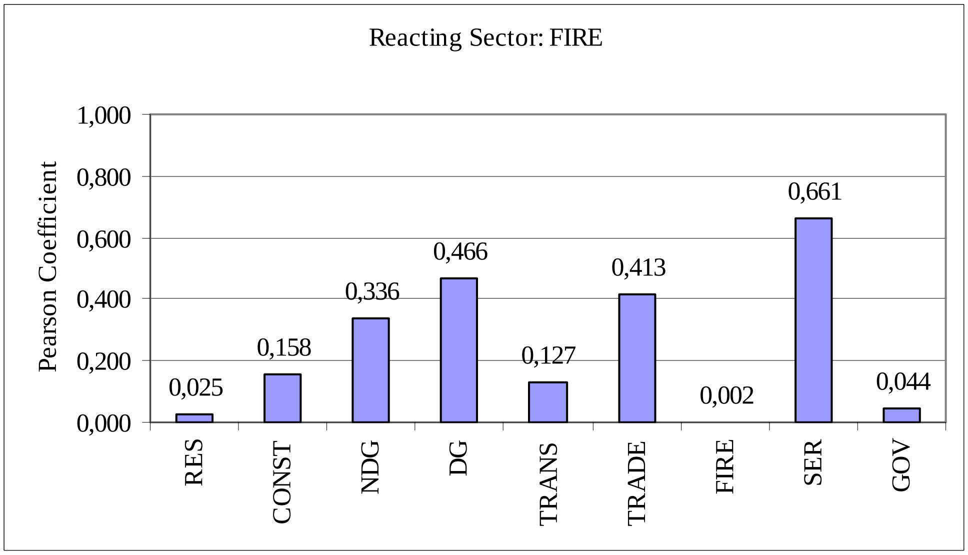
e)


Reacting Sector: Transportation
Pearson Coefficient
1,000
0,800
0,600
0,400
0,200
0,000
0,102
0,194
0,322
0,505
0,294
0,236
0,114
0,667
0,037
RES
CONST
NDG
DG
TRANS
TRADE
FIRE
SER
GOV

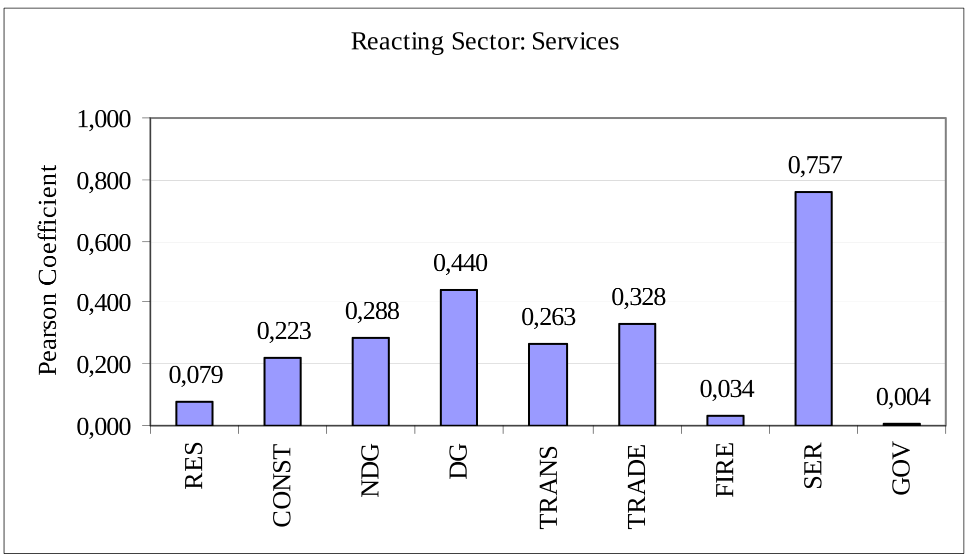
f)


Reacting Sector: Trade
Pearson Coefficient
1,000
0,800
0,600
0,400
0,200
0,000
0,074
0,284
0,335
0,470
0,272
0,387
0,045
0,562
0,020
RES
CONST
NDG
DG
TRANS
TRADE
FIRE
SER
GOV

g)


Reacting Sector: FIRE
Pearson Coefficient
1,000
0,800
0,600
0,400
0,200
0,000
0,025
0,158
0,336
0,466
0,127
0,413
0,002
0,661
0,044
RES
CONST
NDG
DG
TRANS
TRADE
FIRE
SER
GOV

h)


Reacting Sector: Services
Pearson Coefficient
1,000
0,800
0,600
0,400
0,200
0,000
0,079
0,223
0,288
0,440
0,263
0,328
0,034
0,757
0,004
RES
CONST
NDG
DG
TRANS
TRADE
FIRE
SER
GOV

i)

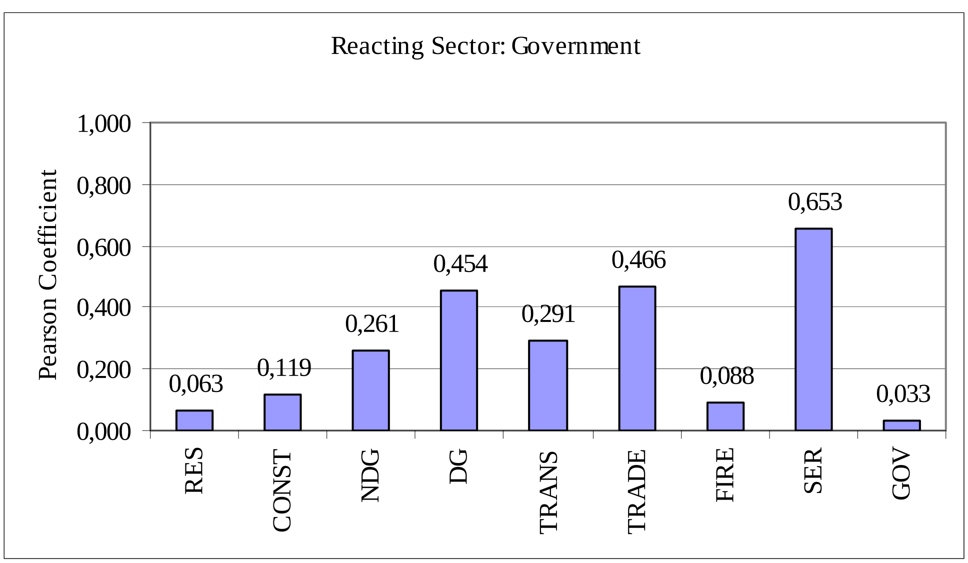

Reacting Sector: Government
Pearson Coefficient
1,000
0,800
0,600
0,400
0,200
0,000
0,063
0,119
0,261
0,454
0,291
0,466
0,088
0,653
0,033
RES
CONST
NDG
DG
TRANS
TRADE
FIRE
SER
GOV

Figure 2a *Backward and Forward Linkages*

Chicago Backward Linkage Hierarchy

Rank	1975	1980	1985	1990	1995	2000	2005	2010
1	4	4	4	4	4	8	8	8
2	3	3	3	8	8	4	3	4
3	8	8	8	3	3	3	4	3
4	5	5	5	2	2	2	2	2
5	6	6	2	5	5	5	5	5
6	2	2	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	9	9	9	9	9	9	9	9
9	1	1	1	1	1	1	1	1

Source: Hewings *et aii* (1998)

Chicago Forward Linkage Hierarchy

Rank	1975	1980	1985	1990	1995	2000	2005	2010
1	9	9	9	9	9	9	9	9
2	2	2	2	1	1	1	1	1
3	6	8	8	6	5	5	4	4
4	8	6	6	2	6	4	5	6
5	5	5	5	8	4	6	6	5
6	4	4	4	5	8	8	8	8
7	3	3	1	4	2	2	2	2
8	7	1	3	3	3	3	3	3
9	1	7	7	7	7	7	7	7

Source: Hewings *et aii* (1998)

Figure 2b *Backward and Forward Simulated Linkages*

Chicago Absolute Importance Hierarchy

Rank	1975	1980	1985	1990	1995	2000	2005	2010
1	4	4	4	4	8	8	8	8
2	3	3	8	8	4	4	3	4
3	8	8	3	3	3	3	4	3
4	5	5	5	2	2	2	2	2
5	6	6	2	5	5	5	5	5
6	2	2	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	9	9	9	9	9	1	1	1
9	1	1	1	1	1	9	9	9

Chicago Absolute Sensitivity Hierarchy

Rank	1975	1980	1985	1990	1995	2000	2005	2010
1	9	9	9	9	1	9	1	1
2	2	2	2	1	9	1	9	9
3	6	8	8	6	5	5	4	4
4	8	6	6	2	6	4	5	6
5	5	5	5	8	4	6	6	5
6	4	4	3	5	2	2	8	8
7	3	3	1	4	8	8	2	2
8	7	1	4	3	7	7	3	3
9	1	7	7	7	3	3	7	7