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Using Average Propagation Lengths to Identify Production Chains in the Andalusian Economy

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Abstract

In this paper, we adopt the viewpoint that not only the size of sectoral linkages is relevant but also the economic distance between sectors. To measure distance, we define the average propagation length as the average number of steps it takes an exogenous change in one sector to affect the value of production in another sector. This distance does not depend on whether the linkages are forward or backward in nature. Combining the size of the linkages and the distance between sectors, allows us to visualize the production structure in terms of production chains. The empirical application studies the Andalusian economy.

Keywords: Input-Output Analysis; Linkages; Production Chains; Andalusia

JEL: C67, D57 and R15

1. Introduction

One of the characteristic features of input-output analysis is that it takes full account of the interdependent nature of the production structure. That is, if sector i buys (some of) its inputs from sector k and sector k buys inputs from sector j , production in sector i depends indirectly on inputs from sector j . This holds for all sectors in the economy so that each sector depends – directly or indirectly – on any other sector. There is a vast body of literature dealing with the question how to measure such interdependencies (or linkages) appropriately. The techniques that have been suggested range from simple measures to highly sophisticated methods. Examples in the first category include the straightforward use of the coefficients of the input matrix (originally proposed by Chenery and Watanabe, 1958) and of the so-called Leontief inverse matrix (developed in Rasmussen, 1956). Examples in the second category include the use of hypothetical extractions (introduced by Strassert, 1968; see Miller and Lahr, 2001, for an overview), qualitative input-output analysis (proposed by Czayka, 1972; see also e.g. Schnabl, 1994), inverse important coefficients (see e.g. Hewings, 1984; Schintke and Stäglin, 1985; Aroche-Reyes, 1996), fields of influence (see e.g. Sonis and Hewings, 1989, 1991, 1992), and eigenvectors (Dietzenbacher, 1992). For concise overviews of the literature, see e.g. Kurz *et al.* (1998, pp. *xix-xxvii*) or Dietzenbacher and Lahr (2001), see Sánchez-Chóliz and Duarte (2003) for an overview of indicators.

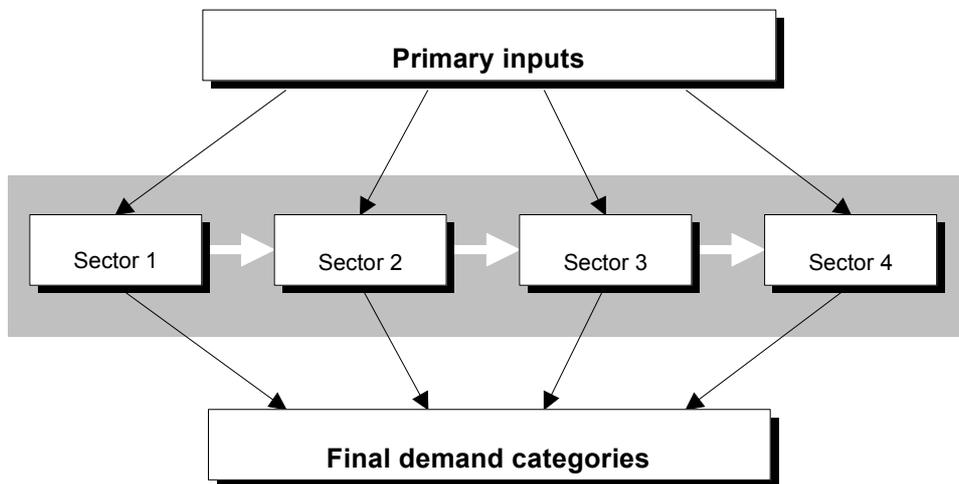
The present paper takes a slightly different viewpoint on linkages. Not only the size of the linkages between two sectors reveals important information, but also the “economic distance” between these two sectors. That is, if sector i largely depends on sector j , it is relevant to know whether this dependence is direct or whether it runs via one other sector, or two (or more) other sectors. When these two elements (i.e. linkage size and distance) are combined, we may visualize the production structure in the form of production chains.

Production chains play an important role in the field of vertical integration, in the discussions on mergers and outsourcing, and in supply-chain management. The central idea goes back to the production theory of the classical Austrian school (e.g. von Böhm-Bawerk, 1921, and Menger, 1923). The production of a good goes through several, successive phases and in each of these phases the combination of intermediate inputs from previous phases with primary inputs (such as labor and capital) adds to the value of the product. Production chains can be well-described at the micro (or firm) level and they have been studied using so-called enterprise input-output analysis (see e.g. Albino *et al.*,

2002, 2003). At the macro level, studies on the triangularization of input-output matrices (introduced by Simpson and Tsukui, 1965) include elements of the Austrian school, in the sense that early phases of production are distinguished from later stages and the final stages.

Let the economy be split into several sectors (or industries) according to methods of production or according to product characteristics. Each industry is assumed to produce a single commodity (which might be a good or service), and uses commodities as intermediate inputs. The produced commodities partly serve in meeting the final demand, but often a major proportion is used in other industries (or the own industry) as an intermediate input. In this way, a commodity may flow through various industries before it reaches its final destination, i.e. the final demand categories (such as consumption, investment and exports). At each stage, value is added to the commodity. This process from the earliest stage in production to the final demand stage is called a *production chain*, and is represented by the gray box in Figure 1. This indicates that, concerning the production chains, we will study the intermediate process only. It should be emphasized that some products of, for example, sector 2 will be sold to consumers. For such final products, the *product* chain is much shorter. Production chains focus on the production process of the economy instead of on single products. Production chains thus cover several product chains.

Figure 1. Prototype example of a production chain



As mentioned before, in order to identify production chains we will develop a method to determine the “economic distance” between two sectors. To this end, the sectoral

intermediate deliveries as published in input-output tables, are transformed into a matrix of average propagation lengths (APLs) between the sectors. Its elements measure the average number of steps it takes a cost-push in industry i to affect the price of product j . Alternatively, the elements can be interpreted so as to measure the average number of steps it takes a demand-pull in industry j to affect the production in sector i . The APLs are developed in Section 3, after a discussion of the background input-output models in Section 2. The method has been applied to the Andalusian economy, the results of which are discussed in Section 4.

2. The input-output background

Suppose the economy is divided into n industries and each industry buys products from and sells products to other industries. Together with primary inputs such as labor, capital and (non-competitive) imports, the purchases from the industries make up the inputs of a certain industry. The output of an industry consists of inter-industrial deliveries and deliveries to final demand categories like export, consumption and investments. The input and output flows can be expressed in an input-output table as in Table 1. Note that all entries are in money terms (e.g. billions of euros).

Table 1. The input-output table

	Industries	Final Demand	Totals
Industries	x_{11} x_{12} ... x_{1n} x_{21} x_{22} ... x_{2n} \vdots \vdots \vdots x_{n1} x_{n2} ... x_{nn}	f_1 f_2 \vdots f_n	x_1 x_2 \vdots x_n
Primary Inputs	w_1 w_2 ... w_n		w
Totals	x_1 x_2 ... x_n	f	

In Table 1, x_{ij} denotes the intermediate deliveries from industry i to industry j , f_i denotes the final demand for products from industry i , x_i gives the output (or production) in industry i , and w_j gives the total use of primary inputs in industry j .

Let \mathbf{e} denote the n -element summation vector, consisting of ones, i.e. $\mathbf{e}' = (1, \dots, 1)$, where vectors are column vectors by definition and a prime is used to indicate

transposition. From the input-output table, the following two accounting equations are obtained.

$$\mathbf{x} = \mathbf{X}\mathbf{e} + \mathbf{f} \quad (1)$$

$$\mathbf{x}' = \mathbf{e}'\mathbf{X} + \mathbf{w}' \quad (2)$$

Define the input coefficients as follows,

$$a_{ij} = x_{ij} / x_j \quad \text{or} \quad \mathbf{A} = \mathbf{X}\hat{\mathbf{x}}^{-1} \quad (3)$$

where $\hat{\mathbf{x}}$ denotes the diagonal matrix with the elements of \mathbf{x} on its main diagonal ($x_i > 0, \forall i$). Equation (1) can now be rewritten as

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{f} \quad (4)$$

This corresponds to the standard Leontief quantity model (see e.g. Miller and Blair, 1985, for a detailed introduction to input-output analysis). Under the assumption that \mathbf{A} is fixed and that all prices remain constant, a change $\Delta\mathbf{f}$ in the final demand quantities affects the production in each sector. Solving the model for this case yields

$$\Delta\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}(\Delta\mathbf{f}) = \mathbf{L}(\Delta\mathbf{f}) \quad (5)$$

where \mathbf{I} denotes the identity matrix and $\mathbf{L} \equiv (\mathbf{I} - \mathbf{A})^{-1}$ the Leontief inverse. Under the common assumptions for solvability of the model, \mathbf{L} can be expressed as a power series.

$$\mathbf{L} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \dots \quad (6)$$

The effects on the output ($\Delta\mathbf{x}$) due to a demand-pull ($\Delta\mathbf{f}$) as given in (5), can be interpreted as a stepwise or round-by-round procedure using (6). The question to be answered is how much the production needs to be increased if final demands increase by $\Delta\mathbf{f}$. The initial effect in round 0 states that $\Delta\mathbf{f}$ itself needs to be produced. In order to produce this additional output, extra intermediate inputs are required directly, amounting to $\mathbf{A}(\Delta\mathbf{f})$ in round 1. Next, these extra intermediate inputs $\mathbf{A}(\Delta\mathbf{f})$ need to be produced

themselves, requiring $\mathbf{A}^2(\Delta\mathbf{f})$ of additional intermediate inputs in round 2. And so forth. The output effects $\Delta\mathbf{x}$ thus consist of an initial effect $\Delta\mathbf{f}$, a direct effect $\mathbf{A}(\Delta\mathbf{f})$, and indirect effects $(\mathbf{A}^2 + \mathbf{A}^3 + \dots)(\Delta\mathbf{f})$.

The typical element l_{ij} of the Leontief inverse \mathbf{L} thus gives the increase of the output (in euros) in industry i , due to a one-euro increase of the final demand in industry j . It reflects the linkage of the buying industry j on the selling industry i , and this dependence on inputs is backward in its nature. That is, it analyzes where the inputs come from that are required to fulfill the demand-pull $\Delta\mathbf{f}$. In contrast to this, forward linkages investigate where the produced output goes to and gives the dependence of the selling industry i on the buying industry j . In developing the forward linkages, the second accounting equation is used.

Define the output coefficients as

$$b_{ij} = x_{ij} / x_i \quad \text{or} \quad \mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{X} \quad (7)$$

The output coefficients b_{ij} give the percentage of the output of industry i that is sold to industry j . Using (7), the accounting equation (2) can be rewritten as

$$\mathbf{x}' = \mathbf{x}'\mathbf{B} + \mathbf{w}' \quad (8)$$

This model is known as Ghosh's (1958) "supply-driven" input-output model. Under the assumption of fixed output coefficients \mathbf{B} , it has been widely used to calculate the changes in output (i.e. $\Delta\mathbf{x}'$) due to a change $\Delta\mathbf{w}'$ as

$$\Delta\mathbf{x}' = \Delta\mathbf{w}'(\mathbf{I} - \mathbf{B})^{-1} = \Delta\mathbf{w}'\mathbf{G} \quad (9)$$

where $\mathbf{G} \equiv (\mathbf{I} - \mathbf{B})^{-1}$ denotes the Ghosh inverse. For a long time, this model was believed to be a quantity model – and as such an alternative to Leontief's "demand-driven" model in (4) – until Oosterhaven (1988) pointed out that an interpretation in terms of quantities is highly implausible. In Dietzenbacher (1997) it was shown that all implausibilities vanish, once the model is interpreted as a price model. As a matter of fact, the Ghosh model in (8) is equivalent to the standard Leontief price model. Where the Leontief quantity model in (4) assumes fixed prices, the Ghosh price model in (8) assumes that all quantities remain constant. Equation (9) then shows the effects of changes $\Delta\mathbf{w}'$ in the primary

costs (recall that the input-output table is in money terms) on the output values $\Delta \mathbf{x}'$. Note that these changes are brought about by passing over the increased costs of production into the prices of the products and thus their output values (since quantities remain constant).

If the primary costs in industry i increase by one euro, the production costs in this industry initially rise by one euro and thus its output value. This is the initial effect. In the first round, the direct effect on industry j is obtained. Since a part b_{ij} of industry i 's output is sold to industry j , and since the output value in industry i is raised by one euro, the production costs in any industry j increase by b_{ij} euros. In the second round, these additional cost increases are passed over to the buyers of the products. This implies that the production costs in any industry k increase by $\sum_j b_{ij} b_{jk}$, reflecting that industry j sells a part b_{ij} to industry k , while the costs in industry j have been increased by b_{ij} in the first round. And so forth. This yields

$$\Delta \mathbf{x}' = \Delta \mathbf{w}'(\mathbf{I} + \mathbf{B} + \mathbf{B}^2 + \mathbf{B}^3 + \dots)$$

Due to a cost-push $\Delta \mathbf{w}'$, the output values change by $\Delta \mathbf{x}'$, which is decomposed into an initial effect $\Delta \mathbf{w}'$, a direct effect $\Delta \mathbf{w}'\mathbf{B}$ in the first round, and indirect effects in subsequent rounds amounting to $\Delta \mathbf{w}'(\mathbf{B}^2 + \mathbf{B}^3 + \dots)$. If we take for $\Delta \mathbf{w}'$ the i -th unit vector, i.e. $\Delta \mathbf{w}' = (0, \dots, 0, 1, 0, \dots, 0)$, the elements of the output matrix \mathbf{B} can be given an interpretation. That is, the element g_{ij} gives the increase in the output value of industry j due to a one-euro increase of the primary costs in industry i .

3. The average propagation lengths

In defining the average propagation length, we analyze how a cost-push or a demand-pull propagates throughout the industries in the economy and cumulates into its final effect. In doing so, we extend a technique proposed in Harthoorn (1988). We neglect the initial effects since they do not depend upon the industrial structure and are not relevant for our analysis.

An initial cost-push in industry i raises the output value in industry j by $g_{ij} - \delta_{ij}$ (neglecting the initial effects). δ_{ij} is the Kronecker delta, i.e. $\delta_{ij} = 1$ if $i = j$, and 0 otherwise. A share $b_{ij} / (g_{ij} - \delta_{ij})$ of this output increase requires only one round. The

share $[\mathbf{B}^2]_{ij}/(g_{ij} - \delta_{ij})$ requires two rounds to get from i to j .¹ That is, these effects go through one other industry k ($=1, \dots, n$), since $[\mathbf{B}^2]_{ij} = \sum_k b_{ik} b_{kj}$. The share $[\mathbf{B}^3]_{ij}/(g_{ij} - \delta_{ij})$ requires three rounds, going through two other industries, since $[\mathbf{B}^3]_{ij} = \sum_k \sum_h b_{ik} b_{kh} b_{hj}$. The share of the total effect that requires k rounds amounts to $[\mathbf{B}^k]_{ij}/(g_{ij} - \delta_{ij})$. Note that the shares are non-negative and add to one.

The average number of rounds required to pass over a cost-push in industry i to industry j yields

$$v_{ij} = \{1b_{ij} + 2[\mathbf{B}^2]_{ij} + 3[\mathbf{B}^3]_{ij} + \dots\}/(g_{ij} - \delta_{ij}) \quad (10)$$

Let the numerator of the right hand side of (10) be denoted by h_{ij} with $\mathbf{H} = \sum_k k\mathbf{B}^k$. Then the terms h_{ij} are easily calculated by using

$$\mathbf{H} \equiv \sum_{k=1}^{\infty} k\mathbf{B}^k = \mathbf{G}(\mathbf{G} - \mathbf{I}).$$

The equality can be shown as follows. Note that $\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1}$ so $(\mathbf{I} - \mathbf{B})(\sum_k k\mathbf{B}^k) = \mathbf{B} + \mathbf{B}^2 + \mathbf{B}^3 + \dots = \mathbf{G} - \mathbf{I}$. Hence, $\mathbf{H} = (\mathbf{I} - \mathbf{B})^{-1}(\mathbf{G} - \mathbf{I}) = \mathbf{G}(\mathbf{G} - \mathbf{I})$. The matrix \mathbf{V} of average propagation lengths is thus defined as

$$v_{ij} = \begin{cases} h_{ij}/(g_{ij} - \delta_{ij}) & \text{if } g_{ij} - \delta_{ij} > 0 \\ 0 & \text{if } g_{ij} - \delta_{ij} = 0 \end{cases} \quad (11)$$

In the same way, we can define the average propagation lengths for a demand-pull. Analyzing how a one-euro final demand increase in industry j affects total output in industry i , we find $a_{ij} + [\mathbf{A}^2]_{ij} + [\mathbf{A}^3]_{ij} + \dots = l_{ij} - \delta_{ij}$. The average propagation length for a demand-pull then yields

$$\{1a_{ij} + 2[\mathbf{A}^2]_{ij} + 3[\mathbf{A}^3]_{ij} + \dots\}/(l_{ij} - \delta_{ij}) \quad (12)$$

¹ $[\mathbf{B}^k]_{ij}$ denotes the element (i,j) of matrix \mathbf{B}^k , which differs from $(b_{ij})^k$, of course.

Note that the input matrix \mathbf{A} and the output matrix \mathbf{B} are related to each other. It follows from (3) and (7) that $\mathbf{A}\hat{\mathbf{x}} = \mathbf{X} = \hat{\mathbf{x}}\mathbf{B}$ or $\mathbf{A} = \hat{\mathbf{x}}\mathbf{B}\hat{\mathbf{x}}^{-1}$, which implies that $a_{ij} = x_i b_{ij} / x_j$. In the same way it follows that $\mathbf{A}^k = \hat{\mathbf{x}}\mathbf{B}^k\hat{\mathbf{x}}^{-1}$ and $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = \hat{\mathbf{x}}(\mathbf{I} - \mathbf{B})^{-1}\hat{\mathbf{x}}^{-1} = \hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{x}}^{-1}$. Hence $[\mathbf{A}^k]_{ij} = x_i [\mathbf{B}^k]_{ij} / x_j$ and $l_{ij} = x_i g_{ij} / x_j$. It is immediately clear that the expression for the average propagation length in (12) is equal to the one in (10) or (11). Consequently, the matrix \mathbf{V} defined by equation (11) gives the average propagation length of a cost-push in industry i to affect industry j , as well as of a demand-pull in industry j to affect industry i . This result is in line with the intuition, in the sense that one would expect the average number of forward steps required to get from industry i to industry j to equal the average number of backward steps required to get from j to i .

4. The empirical results for Andalusia

For our empirical analysis, we have measured the APLs for the input-output tables of Andalusia (Instituto de Estadística de Andalucía, 1999, see also Romero, 2003). The original tables were published in two versions, recording 89 and 30 sectors. For our purposes, the latter table has been further aggregated into 6 sectors. The aim of our empirical application is to test the methodology, by checking whether the results are in line with the intuition. In other words, we would like to sketch a rough picture of the Andalusian production structure. For that purpose, results at a high level of aggregation seem more appropriate. Although the original 89-sector classification provides more detail, it is often the case that the details imply that one cannot see the wood for the trees. A further remark is that it should be noted that we study the interdependencies in the regional production structure. That is, the intermediate deliveries record the flows between Andalusian sectors and, thus, do not include imports from the rest of Spain (or the rest of the world).²

The APL values for the 6-sector classification are given in Table 2. Note that the lowest APL values are found principally on the diagonal, i.e. from a sector to itself. This indicates that this “own-sector” dependence is very direct. That is, one subsector depends on another subsector – within the same sector – essentially through direct sales. The dependence is not brought about via one (or more) other sectors, implying that the

intersectoral feedbacks (see Dietzenbacher and van der Linden, 1997) play only a minor role.

Table 2. APLs for the 6-sector –classification

Sector	Agri	Mining	Manuf	Constr	Trade	Serv	Average
Agri	1.23	2.64	1.24	2.48	2.51	1.88	2.00
Mining	1.41	1.24	1.61	1.76	1.53	1.46	1.50
Manuf	1.36	1.62	1.26	1.42	1.57	1.43	1.44
Constr	1.35	1.75	2.17	1.23	1.51	1.45	1.58
Trade	1.20	1.46	1.59	1.56	1.24	1.28	1.39
Serv	1.77	1.43	1.50	1.69	1.23	1.25	1.48
Average	1.39	1.69	1.56	1.69	1.60	1.46	

Agri = Agriculture, hunting, forestry and fishing; Mining = Mining, energy and water supply; Manuf = Manufacturing; Constr = Construction; Trade = Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods; and Serv = Services.

In analyzing the results in Table 2, recall that each figure has a double interpretation. For example, the APL of 2.64 in row Agri and column Mining indicates the average propagation length of a cost-push (which is directed forward) from agriculture to mining. At the same time, however, it gives the average propagation length of a demand-pull (which is directed backward) from mining to agriculture. So, each figure may be interpreted in two directions. In order to avoid any confusion, we will use the terminology forward APL or backward APL, depending on the type of interpretation. For example, the value 2.64 above gives the forward APL from Agri to Mining or, similarly, the backward APL from Mining to Agri.

Low forward APLs (< 1.30) are found from agriculture to manufacturing; from trade to agriculture; from trade to services; and from services to trade. The highest forward APL values (> 2.00) are those from agriculture to mining; from agriculture to construction; from agriculture to trade; and from construction to manufacturing.

To get a general idea of the role of a certain sector, consider the averages. The column with averages gives the row-average forward APL of a sector. It is the largest for agriculture and the smallest for trade. This shows that the agricultural activities can be viewed as being situated at the beginning of production chains in the regional economy. It should be mentioned that Andalusia has an important agro-food cluster which has

² Many input-output studies have used intermediate deliveries that include the imports. Dietzenbacher *et al.* (2005) show that – in that case – the Leontief inverse can only be interpreted under an unrealistically strong assumption.

connections with the tourism cluster through the sector hotels and restaurants, which depends heavily on agriculture. The forward APLs of agriculture (and its row average) show that the dependencies are quite indirect. Agriculture can be seen as a sector in the early stage of these production chains, with several stages following afterwards. Typically, food processing (as part of the manufacturing sector) is involved in the next stage. For trade, with the smallest average forward APLs, the opposite holds. Trade is principally oriented towards final demand and there are not many important forward linkages from trade (as we will see later). It can thus be situated at the end of the production chains.

The row with averages gives the column-average of the backward APLs of a sector. The largest average values are found for mining (including several other extractive industries) and construction. The smallest average backward APL is observed for agriculture, indicating the absence of long production chains that lead to this sector. As we have seen, agriculture is basically situated at the beginning of regional production chains. Construction is at the heart of a regional cluster that includes essentially its supplying industries. Thus, it has relevant backward linkages and is situated at the end of the corresponding production chains.

Summarizing, a large average forward APL and/or a small average backward APL for a sector, suggest that this sector is at the beginning of a (set of) production chains(s). Small average forward APLs and/or large average backward APLs point at a place near the end of some production chain. The case of mining, however, indicates that we should be careful in focusing only on APLs. Mining has the largest average backward APL but no-one would situate this sector at the end of a production chain. It turns out that this sector has only very small linkages, so that APLs provide little (or no) information. As we have seen, low (resp. high) APL values tell us that the effect from one sector to another is primarily direct (resp. indirect). This holds irrespective of the importance (or the size) of the total effect.

It therefore seems obvious to take APLs into consideration only in cases where the size of the linkages is sufficiently large. This implies that two choices need to be made. First, an appropriate measure for the linkages must be selected and, second, it needs to be defined when a linkage is sufficiently large. In line with the development of the propagation length, our choice for the type of linkage is based on the total size of the effect of a cost-push and the effect of a demand-pull. Neglecting the initial effects (just like we did in Section 3), these effects are given by $\mathbf{G} - \mathbf{I}$ and $\mathbf{L} - \mathbf{I}$, respectively. Instead of using the Leontief inverse for the backward linkages and the Ghosh inverse for the forward effects, we have taken the average. Recall that the propagation length is the

same, no matter whether a cost-push or a demand-pull is considered. Therefore we also measure the linkages by taking both directions into account. It should be stressed that other choices are equally valid and the alternatives that we have considered did not much change the final results. So, the linkages are given by the elements of the matrix **F**, which is defined as follows.

$$\mathbf{F} = \frac{1}{2}[(\mathbf{L} - \mathbf{I}) + (\mathbf{G} - \mathbf{I})] \quad (13)$$

The element f_{ij} gives the size of the linkage and equals the average of the forward effect of a cost-push in sector i on the output in sector j and the backward effect of a demand-pull in sector j on the output in sector i . The results are given in Table 3. Note that the figures in the columns for agriculture and mining are all very small (except for the diagonal element of mining). This implies that these sectors have a negligible backward dependence on other sectors and other sectors have a negligible forward dependence upon agriculture and mining. This confirms our earlier findings that both sectors should be situated at the beginning of the production chains.

Closer inspection of the numbers in Tables 2 and 3 suggests that there seems to be an inverse relationship between APLs and elements f_{ij} . The Pearson correlation coefficient equals -0.47. Hence, lower APL values are, to some extent, associated with larger linkages. This implies that the largest impacts between two sectors are often those that are essentially direct.

Table 3. Linkages between the sectors

Sector	Agri	Mining	Manuf	Constr	Trade	Serv
Agri	0.060	0.003	0.311	0.033	0.007	0.033
Mining	0.058	0.206	0.131	0.077	0.064	0.161
Manuf	0.038	0.013	0.168	0.141	0.025	0.056
Constr	0.016	0.003	0.008	0.215	0.011	0.023
Trade	0.036	0.013	0.038	0.032	0.041	0.059
Serv	0.024	0.052	0.078	0.063	0.132	0.152

The APL indicates the “distance” between two sectors by expressing the average number of steps it takes to transmit a cost-push (or demand-pull) from one sector to the other. The limitation of APLs is that the size, and thus the relevance, of the transmission itself may be negligible. On the other hand, the elements of the matrix **F** indicate the size

of the transmissions and points at the relevance of the linkage between two sectors. The limitation of **F**, however, is that it does not allow to distinguish whether the effect is mainly direct or indirect. In the latter case, the transmission from one sector to an other takes at least two steps. In order to visualize the production structure in terms of production chains, the obvious solution is to combine the two types of indicator.

The procedure is to take APLs into account only if the linkage is sufficiently large, using a threshold value *a*. Further, the APLs are rounded to the nearest integer. From the matrix **V** with APLs and the matrix **F** with linkages we construct a new matrix **S** in the following way.

$$s_{ij} = \begin{cases} \text{int}(v_{ij}) & \text{if } f_{ij} \geq a \\ 0 & \text{if } f_{ij} < a \end{cases} \quad (14)$$

where $\text{int}(v_{ij})$ is used to indicate the nearest integer to which v_{ij} has been rounded. For the calculations with the 6-sector classification, we have used a threshold value $a = 0.060$. The values f_{ij} that are larger than the threshold are printed bold in Table 3. The results for the matrix **S** are shown in Table 4.³

Table 4. The matrix **S**

Sector	Agri	Mining	Manuf	Constr	Trade	Serv
Agri	1	0	1	0	0	0
Mining	0	1	2	2	2	1
Manuf	0	0	1	1	0	0
Constr	0	0	0	1	0	0
Trade	0	0	0	0	0	0
Serv	0	0	1	2	1	1

A graphical representation of the matrix **S** is given by Figure 2. Each arrow represents a relevant linkage and gives the (rounded) APL. Solid arrows have an APL of 1, dotted arrows have an APL of 2. Note that the sector's dependency on itself – which is larger than the threshold in all sectors, except trade – have been left out. Further, it should be emphasized that the arrows indicate the APLs from a forward perspective. For example, the arrow from agriculture to manufacturing indicates the forward dependence

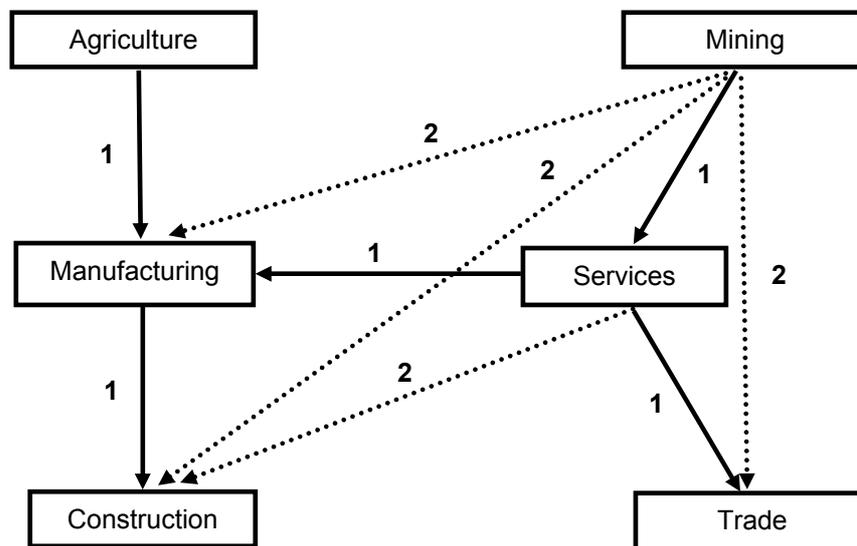
³ The value 1 in the row services and the column manufacturing is based on the exact APL, which is slightly smaller than 1.50. Note that Table 2 gives the rounded numbers.

of agriculture (in transmitting its cost-push) on manufacturing. This is in line with the usual graphs for production chains.

For the moment, let us disregard the (dotted) arrows in Figure 2 with an APL of 2. Then, we can clearly distinguish two production chains. The first is given by agriculture → manufacturing → construction, the second by mining → services → trade. In addition there is a connection between the two chains through the forward dependence of services on manufacturing. Two of the arrows with an APL of 2 are entirely consistent with these chains. The linkage between mining and manufacturing requires two steps, which is in line with the one-step distance between manufacturing and services and between services and mining. Also the two-step dependence of services on construction is in line with the one-step distance between construction and manufacturing and between manufacturing and services. The same holds for the two-step dependence of mining on trade.

However not all arrows with an APL of 2 are entirely consistent. The two-step distance from mining to construction seems to include a “shortcut”, because following the arrows with an APL of 1 requires three steps. Clearly there are many more production chains in the economy, while Figure 2 is based only on the strongest linkages. Several of such routes may be accumulated here, whereas none of them is important in itself.

Figure 2. Production chains in the Andalusian economy



As a second application, we have considered the linkages, APLs and production chains at a more detailed level. For this we have used the 30-sector classification. As mentioned before, the amount of information makes it difficult to see the wood for the trees whenever it is attempted to sketch a picture of the entire economy. Therefore we have chosen to focus the attention on a single sector and its connections. Because of its strategic importance in the regional economy of Andalusia, we will examine agriculture (which includes also hunting and forestry).

Using the full 30-sector input-output table, we have calculated the 30×30 matrices \mathbf{V} with APLs, \mathbf{F} with linkages and \mathbf{S} that lists rounded APLs for the strong linkages. In this case, we have adopted a threshold value of $a = 0.030$. Next we have examined the first row and column (corresponding to agriculture) of the matrix \mathbf{S} . The connections of agriculture – forward in its row and backward in its column – with sector j are indicated by the values of s_{1j} and s_{j1} , respectively. If both are zero, the linkages between agriculture and sector j are not relevant in size and this sector is not taken into account any further. It turns out that agriculture has relevant linkages with eight other sectors (for which s_{1j} and/or s_{j1} is nonzero). The summary of the 30×30 matrix \mathbf{S} restricted to agriculture and its eight linked sectors is given in Table 5.

Table 5. Summary of matrix \mathbf{S} for agriculture and its linked sectors

Sector	1	3	5	8	10	11	19	21	22
1	1	0	1	1	0	0	0	0	2
3	2	1	3	0	0	0	1	3	2
5	0	0	1	0	0	0	0	0	1
8	0	0	0	1	0	0	0	1	0
10	1	0	2	0	1	0	0	2	0
11	1	0	2	0	0	1	0	0	0
19	2	1	2	0	0	2	1	1	1
21	1	0	2	1	0	0	0	1	1
22	0	0	0	0	0	0	0	0	0

1 = Agriculture, hunting and forestry; 3 = Extraction of energy products; 5 = Food, beverages and tobacco; 8 = Wood, wood products and cork (except furniture); 10 = Refined petroleum products; 11 = Chemicals and chemical products; 19 = Electricity, gas and water supply; 21 = Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods; 22 = Hotels and restaurants.

A graphical representation that focuses on the production chains in which agriculture participates is given by Figure 3. Note that the arrows depicted in Figure 3 correspond to the numbers printed in bold in Table 5. In order to keep the picture as simple as possible,

we have graphed only the arrows that begin or end in agriculture and four one-step connections that are relevant for agriculture. One of the first things to notice is that the production chains that involve agriculture are substantially different from the picture sketched by Figure 2. When the entire Andalusian economy was considered, agriculture was found at the beginning of a production chain (and, thus, showing no significant backward linkages). Focusing at a more detailed level on agriculture only, it appears to be in the middle of several chains (with significant backward and forward dependencies). This difference should not come as a surprise. If sector i depends significantly on another sector j , this does not imply that the aggregate sector that includes i also depends significantly on the aggregate sector that includes j . It should also be noted that different threshold values have been used. This is because at a less aggregated level the size of the linkages is typically smaller.

Figure 3. Production chains involving agriculture

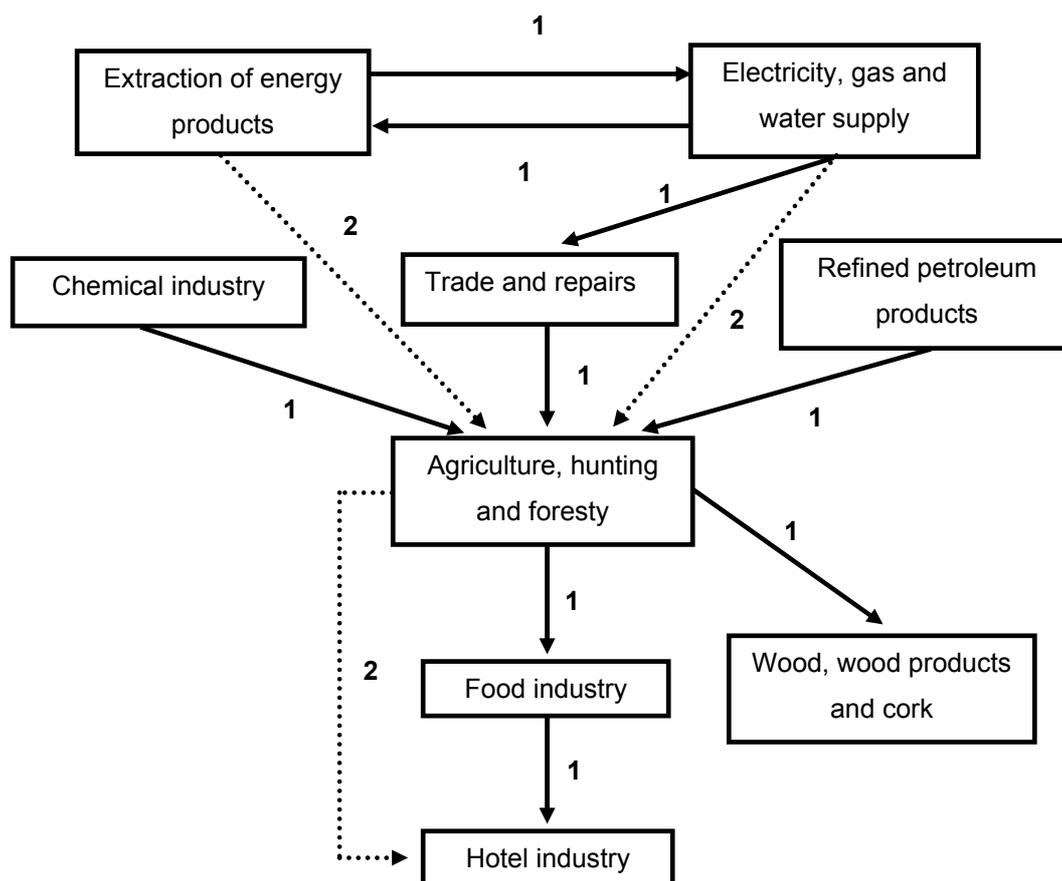


Figure 3 clearly shows the production chains in which agriculture is involved. Note that the two-step forward dependence of agriculture on the hotel industry fits nicely in the picture, because it makes sense that it runs via the food industry (including beverages). At first sight, also the two-step backward dependence of agriculture on electricity, gas and water supply (EGWS) seems to match the picture. In this case, however, we have doubts whether the trade and repairs sector serves as the important intermediate stage. Rather, we believe that the two-step distance between agriculture and EGWS is a cumulative effect involving many chains, all of which are relatively unimportant in themselves. Clearly, the one-step backward dependencies of agriculture on trade and repairs and of trade and repairs on EGWS are part of this accumulated effect. A similar observation can be made for the two-step forward dependence of the extraction of energy products on agriculture, because the shortest route using one-step distances involves three steps.

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