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RETIREMENT EXODUS AND ITS IMPACTS ON REGIONAL ECONOMIES: SIMULATION RESULTS FROM THE CHICAGO CGE MODEL

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Abstract: This paper analyzes the impact of large retirement out-migration (a so called retirement exodus), which is expected to accelerate with the rapid transition to an ageing population over the next two decades. Specifically, this study quantifies the effect of retirement migration and ageing population on both a regional and the national economy in terms of its impact on consumption patterns by age cohort, labor market, and industry structure. For this purpose, this paper uses a two-region Computable General Equilibrium (CGE) model, combined with an Overlapping Generation (OLG) framework, where all the economic interactions arising from demographic changes can be captured. In addition to the macroeconomic consequences of demographic changes, this study will also assess the likely impacts of changes in policy, such as the pension fund liabilities, and favorable foreign immigration policy.

Keywords : regional computable general equilibrium (CGE) model, retirement migration, ageing population

JEL Classification : C68, D58, E62, H55, J14

1. INTRODUCTION

Over the last three decades, Illinois has lost its place as one of the leading states in the national economy. Traditionally, Illinois has been one of the highest per capita income states, ranking among the top ten states with the income much above the national average. However, in 2004, Illinois' per capita income fell to just slightly above the national average, ranking fourteenth highest among the states. This decline has been more serious when economic performance has been viewed in terms of total income rather than per capita. Thirty years ago, the Illinois economy accounted for 6% of national income; by 2004, its share had fallen to 4.5%. Much of this economic downturn is linked to the metropolitan Chicago economy, which plays an important role as a development engine of the state, accounting for over 60% of state population and significant part of state production.

There can be little doubt that the lower level of relative economic performance of both Chicago and Illinois partly resulted from the recessions in the manufacturing sector starting from the early 1980s. Since 1990 alone, the state has lost 231,000 manufacturing jobs at a rate that is almost twice as high as that for the Midwest as a whole. However, the steady downward economic performance over the long term could be convincingly demonstrated by the slow population growth and changing structure of population in this region with a particular increasing concentration of population in the over-65 age group. Although internal (legal and illegal) migration is an increasingly important component of national population change, regional demographic structure is determined by the combination of birth, death, and two types of migration, international and interregional. However, as regional fertility and mortality have become more uniform throughout the United States, migration has become by far the more important factor in changing regional populations. Hence, part of the reason for the slower pace of population growth in Chicago and Illinois might be the outcome of the out-migration of retirees. In actuality, Illinois has experienced heavy loss in elderly populations due to the retirement migration. Since the 1980s, more than 100,000 people age 60 and older in Illinois, which accounts for about 5% of state's cohort of population over 60 years old, out-migrate every 5 years. Considering the in-migrants, in 1985-1990 Illinois has experienced a net loss of 70,000 retirees, who were 60 and older, ranking the second largest among all the states. Further, the Chicago area[1] accounts for over 80% of the older migrants who leave Illinois. As the result, Illinois has lost more than one billion dollars a year in net income through retirement migration since the 1980s (Longino, 1995).

Recent empirical analysis (Serow & Haas, 1992; Day & Barlett 2000) has demonstrated that the large retirement migration will substantially impact the regional economy in terms of consumption, labor market, industry structure, and economic stability. First of all, the retirees who migrate influence local economies primarily through the loss of their consumption demand. The consumption potential of the elderly migrants is relatively much greater than average

[1] The Chicago area is the MSA, comprising the counties of Cook, Will, DuPage, McHenry, Lake, and Kane.

households because they usually have higher disposable income and accumulated assets, providing resources for large purchases. Moreover, they have strong consumption propensities such that they consume almost 90% of their incomes, whereas middle age groups consume only about 70% (Yoon, 2005), thus spend most of their potential income in the local economy, stimulating demand for locally provided goods and services as well as creating new jobs. Therefore, when retiree out-migration occurs, there is a significant contraction in effective demand. In addition to the reduction in total consumption demand, retirement migration has the potential to change the local economic structure through the way in which the withdrawal of local consumption demand affects different industries. For example, retirement out-migration will depress regional service sectors including real estate, financial institutions, health, insurance, and retail, which capture a large slice of the retirees' expenditure. Thirdly, while retirement migration does not affect the labor force in the short run because the vast majority of the retirees do not participate in the labor market, the reduced labor demand (generated by reductions in consumption) will eventually influence the supply of the labor through wage fluctuations. This happens since a reduced labor demand given fixed supply increases the downward pressure on wages, and then relatively lower real wages stimulates workers to leave, further damaging the regional potential production possibilities. Finally, one of the major disadvantages from losing retirees is the loss of economic stability. Most retirees have steady incomes from public pensions, social security benefits, and savings account which are not vulnerable to business cycles in the national economy.[2] Therefore, the loss of retirees may be seen as reducing the structural stability of the regional economy.

The slower pace of development that the Chicago economy has been facing may be related to many factors; at the present time, the impacts of retirement out-migration might only have a modest effect, although recent analysis found that in 2001 the net income effect of all migration (not just retirees) amounted to a net loss of $2 billion to the Chicago economy. However, over the next couple of decades, retiree migration may be expected to have a dramatic impact on the Chicago economy, because of the rapid transition to a status where the ageing population will comprise a larger share of total population than at the present time. This transition is likely to increase the out-flow (a so called retirement exodus). The oldest members of the baby boom

[2] Although income streams based on stock-market portfolios and savings rates will be sensitive to market fluctuations

generation currently are reaching early-retirement age and much higher levels of retirement migration are now a possibility over the next 20 years. Moreover, the percentage of baby boomers who will move across state lines after retirement is expected to increase because the future retiree generation has revealed a stronger tendency to a value better quality of life more highly than earlier generations (Longino, 2003). The transition of baby boomers from work to retirement could also potentially have important perverse socio-economic effects in the future, such as a decline in total savings and per-capita income, and increasing pressures on the cost of public pensions, and intergenerational disparity. The recent press concerning under-funding of both private and public pension funds suggests that this issue is likely to grow in significance over the next two decades.

Taking into account these demographic changes and their socio-economic effects, it is important to evaluate the economic consequences of retirement migration. Since the problem of the retirement migration is not a zero-sum even at the national level, it is important to analyze the effects on the national economy at the same time. Retirees moving into rapidly growing areas create a different set of problems to those in the regions in which they were formerly resident. However, very few studies have investigated the impacts of retirement migration; past attempts have simply compared the difference between retirement and non-retirement areas in terms of economic performance (Serow & Hass, 1992; Bennett, 1993) or analyzed the correlation between elderly migration and major economic indicators (Day & Barlett, 2000). This study would be the first attempt to rigorously quantify the effect of retirement migration and ageing population on the regional and national economies.

This study focuses on a robust analysis of the macroeconomic consequences of demographic changes on both the regional and national economies within a general equilibrium framework. The research proposes to answer the following set of questions:

- How would macroeconomic indicators, such as labor markets, savings, investment and per capita income, vary over time in association with the demographic changes?
- What would happen to savings behavior (asset accumulation) and retirement income corresponding to various pension reforms?
- What are the potential economic and labor market implications of favorable foreign immigration policy?

- What would be the effect of retirement migration on each industry's growth?
- How sensitive will the performance of the regional and national economy be to changes in the way pensions are funded?

To answer to theses questions, the research will use a two-region Computable General Equilibrium (CGE) model, where all the economic interactions arising from demographic changes can be captured since the model contains detailed information about all commodity and factor markets together with each agent's decision making process.

It is hoped the results of this study would present valuable policy implications to local and federal governments to assist them in preparing for the economic and social consequences of future demographic changes. This work should be critically important to those regional states with higher elderly out-migration rates. While the case study will focus on Chicago and the Rest of the US (as the second region), the methodology will be sufficiently portable to make application to other regions possible; future work could extend this structure to a multiregional context.

2. A REGIONAL OVERLAPPING GENERATION MODEL[3]

This model is represented by the two-region dynamic general equilibrium model with an Overlapping Generation (OLG) framework, whose national version was originally developed by Auerbach and Kotlikoff (1987). However, the model has many novel features that differentiate it from other models in its treatment of regional migration, demand structure, and sectoral disaggregation. Theses features are presented in more detail below.

2.1. Regional setup

Even the model economy is composed of two regions, Chicago (HOME) and rest of the US (ROUS), the basic structure of this regional model is closely related to its national counterparts[4].

[3] In this section I present a brief non-technical description of the regional OLG model. A detailed structure of the model is described in the appendix.

Households maximize their utility by choosing a profile of consumption over lifecycle and firms demand factors following from profit maximization, responding to differences in goods and factor prices. Prices adjust in both goods and factor markets to clear the excess demand.

However, this model has a complicated structure, even more than international trade models. This complication arises due to the greater openness between the regions. For example, each region trades in goods not only with foreign countries, but also with the other region. Migration between the regions is easier than between countries. Also, savings in the region are not directly linked to investment in the region.

This model adds various components and linkages into its national version to capture these regional features. First, in this model, labor and financial capital are assumed to be mobile interregionally, with different rates of mobility. Financial capital is perfectly mobile so that there exists a single price for capital. This results in the interest on savings being identical across the regions. Labor is assumed to be partially mobile in domestic regions,[5] while internationally immobile, taking into account people's preference for staying in the region where they originally reside. This locational preference is represented by the wage elasticity of labor migration. With partial mobility of the labor, wage differentials between regions take multiple periods to be adjusted because of the lagged responses of labor supplies.

Secondly, the nesting structure is necessary to complete the household's decision process, since both regions trade in goods and households consider products from different countries and regions as imperfect substitutes (the familiar Armington assumption[6]). A nesting structure is an important element in the model framework, and its hierarchy in this model consists of the following four steps (see figure 1). In the first step, each agent determines the aggregated consumption path over time, maximizing a time-separable utility function subject to lifetime income. Time separability allows a separation between intertemporal and intraperiod decision-making in the nesting structure. Once optimal conditions governing the aggregate consumption levels are established, the next step is to allocate these expenditure levels among the different

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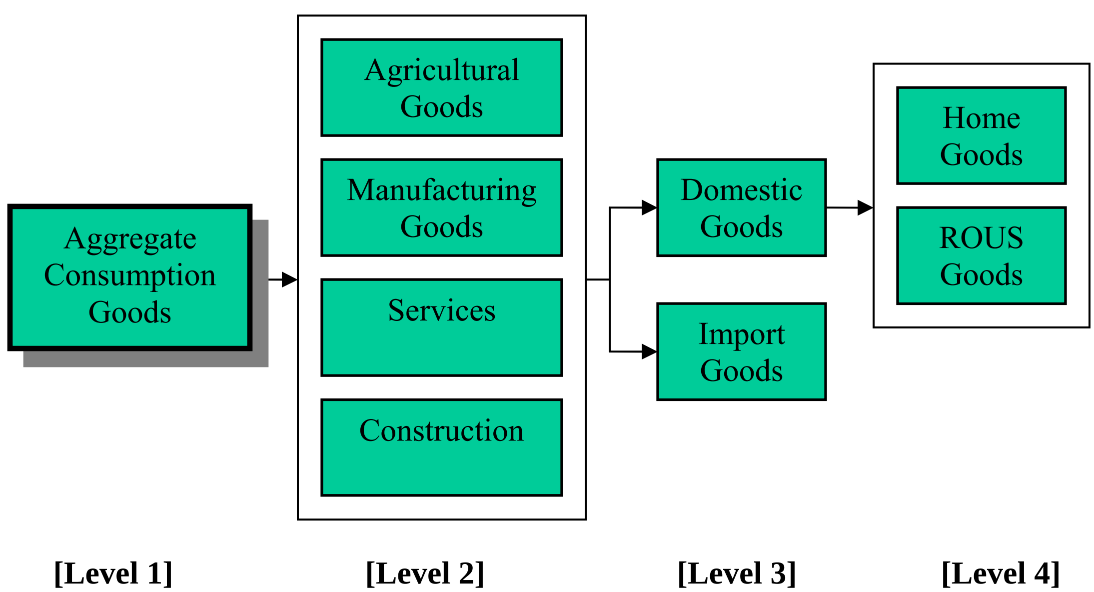
[4] Detailed features of regional CGE modelling can be found in Partridge and Rickman(1998).

[5] According to Jones and Whalley(1986), perfect labor mobility is not useful in analyzing the region specific effect of government policies because under perfect mobility, the policy effect might be underestimated with complete labor movement between regions.

[6] Under Armington assumption, a good produced in one region is treated qualitatively different from good produced in other regions and abroad.

industries. That is, each agent chooses three different final goods from different industries by maximizing its Cobb-Douglas sub-utilities.[7] The next two steps in the household optimization problem are to determine the composition of each industry good in terms of geographic origin. In these two steps, substitution elasticities play an important role in determining each agent's optimal choice. The third step determines the optimal composition of the agent's purchase in terms of interregional origin under constant elasticity of substitution (CES) utilities. That is, US products and imports as substitutes are available for regional consumption. Finally, at the last step, substitution occurs between the goods supplied by the HOME region and ROUS region as in the similar way to the previous step. Elasticity values at the last step are very important to determine the magnitude of the regional effects. For example, even if the aging population changes the age structure in a similar pattern across the nation, the effect on regional economies will depend on theses elasticities.

Figure 1. Nesting Structure in Final Demands for Each Region



[7] Taking into the difficulties in estimating the substitution elasticities between industry goods, this model restricts the utility function as the Cobb-Douglas form whose demand function is defined by only the product of the preference parameter for that good and its relative prices.

2.2. Overlapping generation framework

To measure the effects of the demographic change on the behavior of different generations, it is necessary for the model to be disaggregated by the age cohorts as well as dynamic processes, describing the path of consumption and savings behavior of each age cohort over time. The overlapping generation framework satisfies these criteria. The dynamic model is constructed based on the overlapping generation model developed by Auerbach and Kotlikoff (1987). There are three types of agents in each region: households, firms, and government. Each sector represented by these agents has stylized components, but their interactions can be quite complex. By solving for the economy's general equilibrium transition path, the model takes into account all relevant feedbacks among these agents according to demographic changes and relating government policies.

Household sector

In this model, each region is populated by the households who live up to age 80.[8] The individual agent enters the labor market at the age of 21 and retires at the age of 65. Lifetime uncertainty is not considered in this model. Since all the agents between ages 0 and 20 are considered as children, who are supported by their parents and do not economic activities, this model deals with the agents above age 21. So, there are 12 representative agents in each region, one per age group, with each period corresponding to 5 years. The population growth rate for each cohort is treated as given.

Since all agents in the same age cohort are identical in terms of preferences, individual heterogeneity is present only across age cohorts with respect to labor productivity and asset holdings. Each agent makes lifetime decisions about consumption and savings on the basis of the life cycle model, leaving no bequests and receiving no inheritances. Since each agent is represented as forward looking and having perfect foresight, the evolution of consumption and savings depends on all future interest rates and after tax wages.

The individual life cycle of a representative agent is described by a hump shaped income profile. The agent starts to work at age 21 and receives the highest labor income during his/her middle

[8] Of course, all of these stylized facts can be changed and part of the research agenda will be to consider changes; for example, people are living longer and thus death at 85 or 90 will have enormous implications for the economy.

age. Retirement terminates the flow of labor income and entitles the agent to pension benefits. As a result of the uneven pattern of labor income over their lifetime, agents save during their working period and dissave in retirement. This pattern for the labor income profile reflects the agent's changing productivity over his life cycle. This implies that labor income depends on the individual's productivity, which is assumed to be identical across regions. However, labor income might differ across regions because the wage rate per unit of effective labor is region specific.

Production sector

The representative firm is assumed to produce a unique regional good using Cobb-Douglas type of production technology under perfect competition. The firm hires capital and labor up to the point where factor prices equal their marginal products. Since the economy consists of two regions, capital is the composite good of the two regional final goods. Therefore, the allocation of investment demand between two regions is determined by an investment technology specific to each region. The investment technology is given by a CES function.

Each region contains three different types of industries: agriculture, manufacturing, services.[9] Disaggregating the economy into industries allows for explicit dynamic analysis of the intersectoral reallocation of resources induced by demographic changes.

Government sector

The role of the government in this economy is simply to levy the taxes and administer the social security programs. The government has three types of taxes: wage income tax, consumption tax, and capital tax. Since this economy ignores the public debt, the government balances the budget constraint spending tax revenues without issuing government bonds.

The government also manages the public pension system, which is modeled as a pay-as-you-go scheme for the benchmark economy. The government grants a fixed pension benefit to the

[9] The three produced goods in each region are treated as qualitatively different from goods produced in the other region or abroad (the so-called Armington assumption). The reason for adopting this treatment is the presence of cross hauling in interregional trade, that is, each region can export and import the same good at the same time. At the level of aggregation specified, the degree of intra-industry trade will be very high.

retired generations while pension contributions are financed by the current working generations. The pension benefits are determined as a fraction, the replacement rate, of the lifetime average labor earnings from age 21 through the age of retirement. Since the pension budget constraint is balanced every period, the model can calculate the path of pension contributions from the current working generation, which is endogenously determined.

3. DATA

A key issue in computable general equilibrium modeling is calibration, i.e. the process of selecting values of exogenous parameters to ensure that the solution is consistent with what is observed in the data. The calibration of this model is basically conducted to replicate the equilibrium conditions in the base year. Since national values are easily obtained from the accessible national data set like NIPA and previous studies (Brown, *et al*., 1992; Kouparitsas, 1998) the next section describes the choice of regional parameters.

Steady state conditions and microconsistent data set for Chicago are mostly obtained from the Chicago Social Accounting Matrix (SAM) constructed by MIG (1997), Illinois input-output multipliers and Chicago input-output tables prepared by REAL (Regional Economics Applications Laboratory) in the University of Illinois. Further, a computable general equilibrium model for Chicago has been completed and many of the parameters for this model will be used in the two-region system to be constructed. Most regional elasticities of substitution which appear in the production and utility functions are obtained from the corresponding national counterparts since the model assume identical household preferences and production function across regions. Also, the price elasticities in interregional trade are assumed to be the same as those in international trade following the suggestion by Jones and Whalley (1989). The labor migration elasticity is specified considering the past studies on interregional migration (Plaut, 1981; Seung & Kraybill, 2001).

For the demographic data set, population change by age cohorts until 2050 is obtained from the projections provided by Illinois Department of Commerce and Economic Opportunity. And the evolution of population after 2050 is assumed to stabilize at the level of 2050.

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Appendix : Model Structure

Household Sector

Representative agents of each age group maximize a time separable intertemporal utility function that depends on aggregate consumption goods in each period under intertemporal budget constraint.

$$U_i = \sum_{j=1}^{12} \left(\frac{1}{1+\rho}\right)^{j-1} \frac{(C_{i,j})^{1-\gamma}}{1-\gamma} \tag{1}$$

where $C_{i,j}$ is the consumption of an individual of age group j in i_{th} generation, ρ is the subjective discount rate, γ is the inverse of the intertemporal elasticity of substitution.

Intertemporal budget equation is described as follows :

$$\sum_{j=1}^{9} PV_{i,j}(1-\tau_{w,t}-\tau_{p,t})w_t e_{ij} + \sum_{j=10}^{12} PV_{i,j}\, pen_{i,j} = \sum_{j=1}^{12} PV_{i,j}(1+\tau_{c,t})P_t^C C_{i,j} \tag{2}$$

where PV refers to the factor of present discounted value, $PV_{i,j} = \prod_{k=1}^{j-1}(1+r_{i+j-k})^{-1}$, $\tau_{w,t}$ and $\tau_{c,t}$ are tax rates on labor income and consumption respectively, τ_p is the public pension contribution rate, w_t is the wage at time t, $pen_{i,j}$ stands for the pension benefit of generation i at age group j, and P^C is the price of aggregate consumption good. And $e_{i,j}$ takes into account the labor earning's profile, which is defined as a quadratic function of age(Miles, 1999) where y represents the age.

$$e_{ij} = \lambda_1 + \lambda_2 y - \lambda_3 y^2 \tag{3}$$

Under pay-as-you-go system the pension sector grants a pension to the retirement generations while pension contributions are collected from the working generations. Aggregate pension contribution is determined by the product of the population of working group ($N_{i,j}$), pension contribution rate (τ_p), and labor income.

$$TC_t = \sum_{j=1}^{9} N_{t-j,j}\tau_{p,t} w_t e_{t-j,j} \tag{4}$$

The aggregated pension benefit is determined by the fraction of total labor income. The fraction is given by the replacement rate (ψ), which is assumed to be identical across region.

$$TP_t = \sum_{j=10}^{12} N_{t-j,j}[\psi(\sum_{k=1}^{9} \tau_{p,t} w_t e_{t-k,k})] \quad (5)$$

The time path of the pension contribution rate can be determined endogenously by keeping the budget constraint in public pension sector.

$$TC_t = TP_t \quad (6)$$

With the maximization procedure, the following standard first-order conditions can be derived, concerning consumption per period. Equation (7) implies that the marginal rate of substitution between consuming now and consuming later equals the relative price of consuming later instead of now.

$$C_{i,j} = (\frac{1+r_t}{1+\rho})^{\frac{1}{\gamma}} \{\frac{P_{t-1}^C(1+\tau_{c,t-1})}{P_t^C(1+\tau_{c,t})}\}^{\frac{1}{\gamma}} C_{i,j-1} \quad (7)$$

$$C_t^A = \sum_{j=1}^{12} N_{12-j+1,t} C_{t-j+1,j}$$

where C_t^A is the aggregate consumption at time t, $N_{j,t}$ measures the number of people in age group j at time t.

The following wealth accumulation equation can be obtained with the maximization procedure where $a_{i,j}$ is the asset of generation i at age group j, A_t is the aggregate asset at time t.

$$a_{i,j} = a_{i,j-1}\{1+r_t(1-\tau_{r,t})\} + (1-\tau_{w,t}-\tau_{p,t})w_t e_{i,j} + pen_{i,j} - (1+\tau_{c,t})P_t^C C_{i,j} \quad (8)$$

$$A_t = \sum_{j=1}^{12} N_{12-j+1,t} a_{t-j+1,j}$$

Once these optimal conditions governing the aggregate consumption levels at each period are established, the next step is to allocate these expenditures across the four different final goods by maximizing the agent's preferences represented by a Cobb-Douglas utility function.

$$\max U(c_{i,j,s}) = \prod_s c_{i,j,s}^{\mu_s} \tag{9.1}$$

subject to:

$$P^C C_{i,j} = \sum_s p_s^c c_{i,j,s} \tag{9.2}$$

$$\sum_s \mu_s = 1 \tag{9.3}$$

where $c_{i,j,s}$ is consumption of generation i at age group j for a good produced in industry s, p_s^c is the price of good s, and μ_s is the expenditure share of good s.

Aggregate price (P^C) is determined by a non-linear weighted average of price of good s as follows:

$$P^C = \prod_s \left(\frac{p_s^c}{\mu_s}\right)^{\mu_s} \tag{10}$$

The first order conditions yields the following optimal consumption of a good s.

$$c_{i,j,s} = \mu_s \frac{P^C C_{i,j}}{p_s^c} \tag{11}$$

Once the optimal level of each commodity s consumed is determined, the representative agent of each age group establishes the optimal composition of its purchases in terms of international geographic origin. The preferences of the agent with respect to geographic origin are represented by a constant elasticity of substitution function (CES). The optimal composition of its consumption basket among imports and domestic composite goods in a specific sector s is determined by solving the following optimization problem:

$$\max c_{i,j,s} = [\beta_{US}(c_{i,j,s}^{US})^{\varphi_{US}} + (1-\beta_{US})(c_{i,j,s}^{ROW})^{\varphi_{US}}]^{1/\varphi_{US}} \tag{12}$$

subject to:

$$p_s^c c_{i,j,s} = p_s^{US} c_{i,j,s}^{US} + (1+\tau_{c,s}^{ROW}) p_s^{ROW} c_{i,j,s}^{ROW}$$

where $c_{i,j,s}^{US(ROW)}$ is the consumption of generation i at age group j for US(ROW) produced good, β_{US} is the consumption share parameters for good produced in US, φ_{US} is the parameter that controls taste for variety[10], and $\tau_{c,s}^{ROW}$ refers to the tariff rate on import goods. Optimal consumption of differentiated good between imports and domestic composite takes the following form:

$$c_{i,j,s}^{US(ROW)} = (\varphi_{US})^{\sigma_{US}} [\frac{p_s^c}{(1+\tau_s^{ROW(US)}) p_s^{US(ROW)}}]^{\sigma_{US}} c_{i,j,s} \tag{13}$$

where σ_{US} is the Armington elasticity of substitution for consumption in US between imports and domestic goods.

Combining equation (13) with equation (12) yields the aggregate price p_s^c.

$$p_s^c = [(\varphi_{US})^{\sigma_{US}} (p_s^{US})^{1-\sigma_{US}} + (\varphi_{US})^{\sigma_{US}} ((1+\tau_{c.s}^{ROW}) p_s^{ROW})^{1-\sigma_{US}}]^{1/(1-\sigma_{US})} \tag{14}$$

Once the optimal level of consumption for imports and domestic composite goods is given, the final step is to distribute the optimal consumption of its purchases in terms of domestic geographic distribution. The representative agent of each age group maximizes CES aggregate consumption, which represents the preferences over regional produced goods, given aggregate level of domestic consumption goods.

$$\max c_{i,j,s}^{US} = [\beta_H (c_{i,j,s}^{HOME})^{\varphi_H} + (1-\beta_H)(c_{i,j,s}^{ROUS})^{\varphi_H}]^{1/\varphi_H} \tag{15}$$

subject to :

$$p_s^{US} c_{i,j,s}^{US} = p_s^{HOME} c_{i,j,s}^{HOME} + p_s^{ROUS} c_{i,j,s}^{ROUS}$$

where $c_{i,j,s}^{HOME(ROUS)}$ is the consumption of generation i at age group j for a Chicago (HOME) or rest of the US (ROUS) produced good, β_H is the consumption share parameter for good produced in Chicago region, and φ_H is the parameter that controls taste for variety. Similarly, optimal allocation of consumption expenditure across domestic regional goods and aggregate domestic price is given by as follows:

$$c_{i,j,s}^{HOME(ROUS)} = \varphi^{\sigma_H} [\frac{p_s^{US}}{p_s^{HOME(ROUS)}}]^{\sigma_H} c_{i,j,s}^{US} \tag{16}$$

[10] As long as φ is less than 1, which implies σ is finite, consumers regard each good produced by different origin as imperfect substitutes and prefer variety.

$$p_s^{US} = \{(\varphi_H)^{\sigma_H}[p_s^{HOME}]^{1-\sigma_H} + (\varphi_H)^{\sigma_H}[p_s^{ROUS}]^{1-\sigma_H}\}^{1/(1-\sigma_H)} \tag{17}$$

Production Sector

The representative firm operates with constant returns to scale technologies which is represented by the Cobb-Douglas production function where Y is the output, A and α stand for scale parameter and capital income share respectively, and K and L_e represent the capital stock and effective labor force respectively.

$$Y_t = AK_t^{\alpha} L_{e,t}^{1-\alpha} \tag{18}$$

Capital stock is determined by the following law of motion where I is the investment, δ stands for depreciation rate of capital:

$$K_t = I_t + (1-\delta)K_{t-1} \tag{19}$$

The firm decides the demand for capital (K) and effective labor (L_e) to maximize its profit with the given factor prices of wages (w) and rent (re), which are determined in perfect competitive markets.

$$\frac{re_t}{p_t} = \alpha AK_t^{\alpha-1} L_{e,t}^{1-\alpha} \tag{20}$$

$$\frac{w_t}{p_t} = (1-\alpha)AK_t^{\alpha} L_{e,t}^{-\alpha} \tag{21}$$

Output is produced as differentiated products for sale in the domestic and international markets. Each firm allocates its output between domestic and export markets so as to maximize revenue, subject to the constant elasticity of transformation (CET) function.

$$\max\ p_t^{US} D_t^{US} + p_t^{ROW} E_t^{ROW} \tag{22}$$

subject to:

$$Y_t = [\theta(D_t^{US})^{\xi} + (1-\theta)(E_t^{ROW})^{\xi}]^{1/\xi}$$

where D_t^{US} and E_t^{ROW} are supply for the domestic demand and export respectively, θ is the share parameter, and ζ is the output substitution parameter.

The first order condition yields the shares of sale at domestic and abroad which is determined by relative prices.

$$\frac{D_t^{US}}{E_t^{ROW}} = [(\frac{(1-\theta)}{\theta})(\frac{P_c^{ROW}}{P_c^{US}})]^{-\sigma_{CET}} \tag{23}$$

Government Sector

The government taxes labor income, financial capital income, and consumption expenditures to finance its public expenditure. The government decides tax rates according to budget constraints to balance for each period. The government budget constraint is defined as follows:

$$\sum_{j=1}^{12} N_{j,t}(\tau_{w,t} w_t e_{j,t} + \tau_{c,t} P_t^C C_{j,t} + \tau_{r,t} A_{j,t}) = P_t^C G_t \tag{24}$$

where G_t is the government expenditures at time t.

Migration

Working population in age group from 1 to 9 is assumed to be partially mobile across domestic regions but perfectly mobile across industries within a region. The net out-migration of labor is determined by the wage elasticity of labor migration.

$$M_t^w = POP_t^w (1 - \frac{w_t^{HOME}}{w_t^{ROUS}})^\eta \tag{25}$$

where M_t^w denotes the number of net out-migration of labor at time t, POP_t^w is the aggregate stock of labor given at the beginning of time t, w_t^{HOME} and w_t^{ROUS} are the wage rates in Chicago (HOME) and the rest of the US (ROUS), and η refers to the wage elasticity of labour migration.

The stock of effective labor is defined as the number of net workers (N_t^w) times their corresponding productivity level (e_j).

$$\begin{aligned} L_{e,t} &= \sum_{j=1}^{9}(POP_{j,t}^w - M_{j,t}^w)e_{j,t} \\ &= \sum_{j=1}^{9} N_{j,t}^w \cdot e_{j,t} \end{aligned} \tag{26}$$

Retirees aged over 65 are assumed to migrate from one region to the other region with exogenously given rate ε where M_t^R is the number of migrants at time t.

$$M_t^R = \varepsilon \cdot POP_t^R \tag{27}$$

Market Clearing Conditions

There are two equilibrium conditions that close the model. First, the equilibrium conditions for four goods markets must hold, which states that domestic output is equal to total demand from household (C_t), government (G_t), and firms (I_t).

$$\begin{aligned} D_t^{US} &= C_t + G_t + I_t \\ &= [\sum_{j=1}^{12}(N_{j,t}^{HOME} \cdot C_{j,t}^{HOME} + N_{j,t}^{ROUS} \cdot C_{j,t}^{ROUS})] + G_t + K_t - (1-\delta)K_{t-1} \end{aligned} \tag{28}$$

The second condition is equilibrium in the financial market. Financial market equilibrium condition assures that the sum of the stock of wealth accumulated by the all household must be equal to the sum of the stocks of capital used in each region.

$$A_t^{HOME} + A_t^{ROUS} = K_t^{HOME} + K_t^{ROUS} \tag{29}$$