INDUSTRIAL CLUSTERS IN THE INPUT-OUTPUT ECONOMIC SYSTEM

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REAL 07-T-1 January, 2007

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Abstract. The topological principles of the well-known Atkin Q-analysis are applied to the identification of clusters of industries using input-output systems. The operational methodology of Q-analysis is presented in detail and supported by empirical application to the analysis of the Chicago economy in 2000. The central point of the paper is the interpretation of the structural chains of highest dimension as the most significant input-output industrial clusters. This new methodology provides a new way for visualizing economic complexity through the process of structural economic complication.

I. Introduction

This chapter returns to the issue of cluster identification using a set of interindustry accounts; in this sense, it is rooted more in the legacy of industrial cluster and complex analysis associated with the early work of Czamanksi (1971, 1974, 1976) and Czamanski and Ablas (1979) and deepens the now more involved cluster based development strategies described in detail by Bergman and Feser (2000) and the methods linking clusters and innovation presented in Bröcker *et al*. (2003). It does not focus on the more extensive cluster based approaches popularized by Porter (1990) since the objective is to explore the industrial interdependencies in more detail. However, it does share with Dridi and Hewings (2002) the need to make more imaginative use of the structures present in interindustry tables to draw out more information about the structure of the economy being evaluated.

The major purpose of this chapter is to propose a new method of identification of the more important industrial (sectoral) backward and forward linkages clusters in input-output systems in a way that avoids the rigidities of some of the earlier approaches (that identified mutually exclusive clusters). Our attention is directed to the application and further elaboration of the ideas of combinatorial topology to the analysis of economic structure of input-output systems in the form of structural Q-analysis originally proposed by Atkin (1974, 1981) for the analysis of the structure of human interactions. Our central concern is the *complication* of regional or interregional structure that results from the deepening of economic complexity in the form of hierarchies of interacting economic subsystems. Industrial clusters are thus seen as important examples of such subsystems. Their structural changes will require new tools for illustration, interpretation and visualization. We will start from the presentation and the interpretation of the procedure of structural Q-analysis based on the *slicing* procedure of the ordered set of the elements of the Leontief inverse. Further, the chains of structural complication and rank-size ordering procedure will be introduced and interpreted as backward and forward industrial linkages clusters.

An important component of the modern process of industrialization is the change in the nature of interdependence in production characterized by the essential interdependence found in input-output and social accounting tables. Analysis of the *evolution* of interindustry relations has now become, once more, a major point of interest for economic analysts. The traditional approach, proposed by Chenery in the 1950s (Chenery, 1953; Chenery and Watanabe, 1958; Chenery and Clark, 1959) was extended further in various subsequent studies (see Carter, 1970; Long Jr., 1970; Ohkawa and Rosovsky, 1973; Song, 1977; Matthews *et al.*, 1982; Harrigan *et. al.*, 1980; Deutsch and Syrquin, 1989 among others). The main purpose of this chapter is to illustrate some new approaches using Q-analysis to enhance the understanding of the economic structural changes caused by simultaneous technological changes reflected in a set of input-output tables. With this methodology, alternative slicing procedures can be adopted to reveal the finer structure of an economy. In addition, the methodology may be seen to have important relationships with popular notions of backward and forward linkages.

In the next section, the methodology will be described; section 3 develops the slicing procedure that is derived from the decomposition algorithm. This section also provides an illustration with reference to the Chicago metropolitan region for the year 2000. Section 4 presents the industrial clusters and their augmentation. The paper concludes with some summary comments and potential links to some recent work proposing the notion of fragmentation of production systems.

II. Methodology of Structural Q-analysis.

The following methodological description of the procedure of Q-analysis is taken from the Atkin studies (Atkin, 1974, 1981; see also, Sonis, 1988, Sonis and Hewings, 1998, 2000; Sonis, *et al*., 1994).

II.1. Slicing procedure.

Consider the Leontief inverse matrix $B = \left\| b_{ij} \right\|$ of some Input-Output system and let $(i_1, j_1), (i_2, j_2), ..., (i_m, j_m)$ be a fixed set of pairs of economic sectors entering the input-output system. Let $b_{i_1 j_1}, b_{i_2 j_2}, \ldots, b_{i_m j_m}$ be the corresponding components of the matrix *B*. The *slicing procedure* results in the construction of a new matrix B_s whose only non-zero components are $b_{i_1 j_1}, b_{i_2 j_2}, \ldots, b_{i_m j_m}$ while all other components are zeroes. This slicing procedure referred to as a variable *filter* approach is the basic element of minimal flow analysis (see, Holub and Schnabl, 1985, Holub, *et al*., 1985; Schnabl and Holub, 1979 and Schnabl, 1993).

The matrix I_S with the unit entries on the place of non-zero components of the matrix B_s is called the *incidence matrix* associated with the slicing procedure. Obviously, 2^{n^2} different slicing procedures exist for each *nxn* matrix *B*. The simplest slicing procedure consists of the choice of the slicing parameter μ, and the exclusion from the matrix *B* of all components b_{ij} such that $b_{ij} < \mu$. The choice of a definite slicing parameter depends on the investigator's preferences about the economic structure of the interaction matrix.

II.2. Simplicial families for backward linkages.

We will consider the procedure of the Q-analysis of backward linkages (forward linkages can be considered analogously). Consider a slicing procedure defined with the help of the set of components, $b_{i_1 j_1}, b_{i_2 j_2}, \ldots, b_{i_m j_m}$. This procedure defines the sliced matrix B_s and the corresponding incidence matrix I_S. The set $j_1, j_2, ..., j_m$ of the corresponding economic sectors serves as a set of vertices of a many-dimensional polyhedron generating the *partial backward linkages backcloth***.**

The procedure for the construction and partition of this polyhedron into a set of simplexes can be defined in a following way: for each fixed economic sector, $i_k, k = 1, 2, ..., m,$ consider the set of all different economic sectors $j_{r_0}, j_{r_1}, ... j_{r_{qk}}$ corresponding to the non-zero $b_{i_k j_{r_0}}, b_{i_k j_{r_1}}, ..., b_{i_k j_{r_{qk}}}$ associated with the inputs into the sector i_k from the economic sectors $j_{r_0}, j_{r_1}, ... j_{r_{qk}}$. The *simplex*, $S^b_{q_k}(i_k) = S_{i_k}$ associated with the sector i_k, is a minimal convex polyhedron in q_k-dimensional space with *q+1* vertices $j_{r_0}, j_{r_1}, ... j_{r_{qk}}$.

<<insert figure 1 here>>

Figure 1 provides an example of an incidence matrix from a 10x10 input-output table; sales (rows) are shown as *S*(*i*) entries and purchases (columns) as *P*(*j*) entries; interaction between sectors is signified by a value "1" while a "0" indicates no interaction. From this matrix, the simplex associated with S1 (5-simplex) and P10 (4-simplex) are shown as illustrations.

The set of simplexes, $S^b_{q_1}(i_1), S^b_{q_2}(i_2), ..., S^b_{q_m}(i_m)$ associated with all economic sectors i_k, *k=1,2,...,m*, is called *the backward linkages simplicial family*, generating the polyhedron with vertices $j_1, j_2, ..., j_m$, and its partition - *the simplicial complex K(S).*

II.3. q-nearness and q-connectedness.

Two simplices $S(i_k)$ and $S(i_s)$ are *q-near* in the simplicial family *iff* they share at least *q+1* vertices. Thus, two sectors i_k and i_s are *q-near iff* there are at least *q+1* economic sectors with the inputs into the sectors i_k and i_s. If all vertices of a simplex $S(i_s)$ are the vertices of a simplex $S(i_k)$ then the simplex $S(i_s)$ is a face of the simplex $S(i_k)$. In figure 1, the shared face is between the simplexes in highlighted (P3, P8 and P9); from this the complex where by these sets of activities are liked (through the shared face) is also shown.

Two simplices $S(i_k)$ and $S(i_s)$ are *q-connected* by a chain of simplices of length *r iff* there is a sequence of *r* pair-wise *q-near* simplices $S(i_k), S(i_p), ..., S(i_q), S(i_s),$. The relationship *of q-connectedness* generates the partition of the simplicial family *K(S)* into *q-connected* components. The enumeration of all *q-connected* components for each dimension $q \geq 0$ is the essence of the Q-analysis of the simplicial family. In figure 1, two simplices are connected by a shared face (P3, P8 and P9) generating a complex.

I1.4 Procedure and meaning of the backward linkages Q-analysis.

Following Atkin (1974, 1981) the operational basis for Q-analysis is given by ***a shared face matrix*** *SF* of the form:

$$SF = I_S I_S^T - U \tag{1}$$

where I_S is the incidence matrix corresponding to the chosen slicing procedure, I_S^T is its transpose and *U* is the matrix with unit entries. The components of the matrix *SF* provide the amounts of mutual vertices for each pair of sectors i_k, i_s, $k, s = 1, 2, ..., m$. In other words, the components of the shared face matrix *SF* are the dimensions of the maximal mutual faces for each pair of simplices $S(i_k)$, and $S(i_s)$.

The Atkin operational algorithm for Q-analysis includes the following iterative steps for each dimension *q, q = 0,1,...,N*, where *N* is the maximal dimension of simplices from the simplicial complex:

Identify the economic sectors and their corresponding simplices whose dimensions are equal to or larger than *q*; these dimensions are on the main diagonal of the shared face matrix *SF*.

Identify all distinct *q-connected* components - *q-chains* - of the set of simplices constructed in the previous step: two *q-dimensional* simplices $S_q(i_k)$ and $S_q(i_s)$ belong to the same *q-chain* if the corresponding rows i_k and i_s of the shared face matrix *SF* include at least one column with entries larger than or equal to *q*; the number of distinct *q-chains* is denoted as Q_q. The vector

$$Q = \{Q_N, Q_{N-1}, \ldots, Q_o\} \tag{2}$$

is called the *structural vector* of the simplicial complex *K(S)* and the maximal *q-value N* is a dimension of this complex.

II.5. Chains of structural complication of simplicial families and the rank-size ordering.

Consider two slicing procedures S_1 and S_2 and their corresponding simplicial families associated with the simplicial complexes $K(S_1)$ and $K(S_2)$. The simplicial complex $K(S_2)$ is called the *structural complication* of the simplicial complex $K(S_1)$ and noted $K(S_1) \prec K(S_2)$ if each simplex $S_p^{/}(i_k)$ from $K(S_1)$ is a face of some simplex $S_q''(i_k)$ from $K(S_2)$. This means that the incidence matrix I_{S_2} includes all non-zero (unit) components from the incidence matrix I_{S_1}. The set of *m* simplicial complexes $K(S_1), K(S_2), \ldots, K(S_m)$ is called *the chain of structural complication* if for each pair of complexes $K(S_s)$ and $K(S_r)$, one of them is the structural complication of the other. Obviously, the chain of structural complication is defined with the help of the set of corresponding incidence matrices such that, for each pair of incidence matrices, one of them includes all the units from the other. This means also that the chain of structural complication is generated by the sequence of extending sets of the components of the interaction matrix *B*.

One of the important methods of the generation of the chains of structural complication will now be illustrated, namely, the *rank-size ordering method*. The rank-size ordering method is based on the construction of the sequence of all components of the interaction matrix $B = \left\| b_{ij} \right\|$ ordered by size in such a way that the largest components are at the top of the decreasing-by-size sequence of components. Thus, it is possible to consider only the qualitative rank-size sequence in place of the absolute value of each component. Consider the sequence of slicing procedures and the corresponding set of sliced matrices, the first of which includes only the largest components of the interaction matrix *B*, while the second matrix includes the two largest components, the third matrix includes three largest components, and so forth. In such a way, one obtains the chain of structural complication associated with the relative size on the matrix components. The Q-analysis of each element of the chain of structural complication reveals some hidden features of the intersectoral interactions.

III. Slicing procedure based on Decomposition method: Links between Q-analysis and the Superposition Principle[1]

The main operational tool of Q-analysis is the slicing procedure, i.e. a procedure for the choice of unit non zero components in the matrix of structural incidence. Simultaneously, the algorithmic procedure of the superposition principle (see, Sonis and Hewings, 1988, 1998, 2000) generates the decomposition of the flow matrix *A* into the weighted sum:

$$A = p_1 A_1 + p_2 A_2 + ... + p_k A_k \tag{4}$$

where the components $p_i A_i$, $i = 1,2,...,k$, represent the hierarchy of spatio-economic sub-structures within the input-output or social accounting system. Since each matrix A_i is the optimal solution of some linear programming optimization problem, usually it includes many zero components. We can use the decomposition (4) for derivation of the set of incidence matrices for Q-analysis of the flow matrix *A* in the following way. First consider the first tendency $p_1 A_1$ and construct the first incidence matrix I_S^1 whose unit non-zero components are located in the place of non-zero components of matrix, $p_1 A_1$. The first incidence matrix I_S^1,

[1] This section draws on Guo *et. al,* (2005)

represents the first slicing for the matrix *A*. The second slicing will be associated with the incident matrix I_S^2 including the unit non-zero components located in the place of non-zero components of the matrix, $p_1A_1 + p_2A_2$. In the same way the incident matrix, I_S^r, will be generated with the help of non-zero components of the matrix $p_1A_1 + p_2A_2 + ... + p_rA_r; r = 1, 2, ..., k$.

Next, for the description of the structural complication of the flow, *A*, we can apply the procedure of Q-analysis to each incidence matrix from the sequence of incident matrices $I_S^1, I_S^2, ..., I_S^k$. The corresponding sequence of the structural vectors

$$Q^r = \left\{Q_N^r, Q_{N-1}^r, ..., Q_0^r\right\}, r = 1, 2, ..., k \tag{5}$$

will present the topological structural complication of the matrix of flows *A*. The component, Q_N^r, will be interpreted further as the main backward linkages industrial cluster.

III.1. Example

For better demonstration of the slicing procedure of the construction of incidence matrix corresponding to the extreme tendencies $p_1A_1 + p_2A_2 + ... + p_rA_r; r = 1, 2, ..., k$, we consider the case of the analysis of the main backward linkages industrial clusters of the Chicago metropolitan economy in 2000. Table 1 describes the aggregated sectors used in the analysis of the Chicago economy; the input-output tables are extracted from the region's econometric-input-output model (see Israilevich *et al*., 1997 for details) while the flows are shown in table 2.

<<insert tables 1,2 and 3 here>>

Table 3 defines the first extreme tendency acting within the framework of the backward linkages.

The first incident matrix, corresponding to this first extreme tendency has a form

$$p_1 A_1 = 0.319\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

This implies that first incidence matrix is as follows:

$$\Lambda^1 = I_S^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The second slicing will have a form

$$p_1 A_1 + p_2 A_2 = 0.319\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + 0.248\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

This generates the second cumulative incidence matrix. This cumulative incidence matrix includes units in the place they appeared in one of the matrices A_1, A_2:

$$\Lambda^2 = I_S^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The next incidence matrices can be constructed in the same manner:

$$p_1A_1 + p_2A_2 + p_3A_3 =$$

$$= 0.319\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + 0.248\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + 0.166\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

with cumulative incidence matrix generating from matrices A_1, A_2, A_3:

$$\Lambda^3 = I_S^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$p_1A_1 + p_2A_2 + p_3A_3 + p_4A_4 =$$

$$= 0.319\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + 0.248\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + 0.166\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + 0.103\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

with

$$\Lambda^4 = I_S^4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

III.2. Structure of the simplices of backward linkages for each industry.

The augmentation of structure of the simplices of backward linkages for each industry can be extracted from the location of units in the set of columns of matrices $\Lambda^1, \Lambda^2, \Lambda^3, \Lambda^4$ corresponding to the given industry (see table 4).

<<insert table 4 here>>

Table 4 describes the augmentation of the structure of the backward simplices for each industry *S(RES),S(CNS), S(MNF), S(TTF), S(SRV)* and *S(GOV)* with the help of the sequence of four columns corresponding to industries in the incidences . $\Lambda^1, \Lambda^2, \Lambda^3, \Lambda^4$

III.3.Analysis of the shared face matrices for cumulative incidence matrices

The numerical procedure of structural Q-analysis yields the following shared face matrices SF^1, SF^2, SF^3, SF^4, that correspond to the sequence of the incidence matrices $\Lambda^1, \Lambda^2, \Lambda^3, \Lambda^4$:

$$SF^1 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix}$$

with q-chain

$$Q_q^1 = \begin{cases} q = 4 : \{MNF\} \\ q = 3 : \{MNF\} \\ q = 2 : \{MNF\} \\ q = 1 : \{MNF\} \\ q = 0 : \{MNF\}, \{TTF\} \end{cases} \tag{6}$$

and the structural vector $Q^1 = \begin{Bmatrix} 4 & & & & 0 \\ 1 & 1 & 1 & 1 & 2 \end{Bmatrix}$;

$$SF^2 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 0 & -1 & 0 & -1 & -1 \\ -1 & -1 & 4 & 4 & -1 & -1 \\ -1 & 0 & 4 & 5 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix}$$

with the structural chain:

$$Q_q^2 = \begin{cases} q = 5 : \{TTF\} \\ q = 4; \{MNF, TTF\} \\ q = 3; \{MNF, TTF\} \\ q = 2; \{MNF, TTF\} \\ q = 1; \{MNF, TTF\} \\ q = 0; \{MNF, TTF, CNS\} \end{cases} \tag{7}$$

and the structural vector $Q^2 = \begin{Bmatrix} 5 & & & & & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{Bmatrix}$;

Further,

$$SF^{3}=\begin{pmatrix}-1&-1&-1&-1&-1&-1\\-1&1&1&1&-1&-1\\-1&1&5&5&3&-1\\-1&1&5&5&3&-1\\-1&-1&3&3&3&-1\\-1&-1&-1&-1&-1&-1\end{pmatrix}1$$

with structural chain

$$Q_q^3=\begin{cases}q=5:\{MNF,TTF\}\\q=4;\{MNF,TTF\}\\q=3;\{MNF,TTF,SRV\}\\q=2;\{MNF,TTF,SRV\}\\q=1;\{MNF,TTF,SRV,CNF\}\\q=0;\{MNF,TTF,SRV\}\end{cases}\tag{8}$$

and the structural vector $Q^{3}=\begin{Bmatrix}5&&&&&0\\1&1&1&1&1&1\end{Bmatrix}$;

Next

$$SF^{4}=\begin{pmatrix}-1&-1&-1&-1&-1&-1\\-1&3&3&3&3&-1\\-1&3&5&5&5&-1\\-1&3&5&5&5&-1\\-1&3&5&5&5&-1\\-1&-1&-1&-1&-1&-1\end{pmatrix}$$

with structural chain

$$Q_q^4 = \begin{cases} q = 5: \{MNF, TTF, SRV\} \\ q = 4; \{MNF, TTF, SRV\} \\ q = 3; \{MNF, TTF, SRV, CNS\} \\ q = 2; \{MNF, TTF, SRV, CNS\} \\ q = 1; \{MNF, TTF, SRV, CNS\} \\ q = 0; \{MNF, TTF, SRV, CNS\} \end{cases} \tag{9}$$

and the structural vector $Q^4 = \begin{Bmatrix} 5 & & & & & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{Bmatrix}$

IV. Industrial clusters and their augmentation

The consideration of the first incidence matrix Λ^1 and its structural vector (see equation 7) will generate the appearance of the main industrial cluster in the first decomposition tendency. The chain of the highest dimension $q = 3: \{MNF\}$ presents the main industrial cluster existing in the first tendency of the decomposition of the matrix of backward linkages. The structure of this cluster can be derived from the cumulative incidence matrix corresponding to the first tendency:

$$\Lambda^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \overset{\substack{\textit{Cluster} \\ \downarrow}}{\{MNF\}} = \left\{ \begin{array}{|c|c|c|c|c|c|} \hline RES & CNS & MNF & \quad & SRV & \quad \\ \hline \end{array} \right\}$$

The appearance of the next industrial cluster of the highest dimension $q = 5: \{TTF\}$ can be derived from the second incidence matrix (see equation 8):

$$\Lambda^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} \quad \mathit{Cluster} \\ \qquad \downarrow \\ \\ \to \{\ TTF\ \} = \left\{ \begin{array}{|c|c|c|c|c|c|} \hline RES & CNS & MNF & TTF & SRV & GOV \\ \hline \end{array} \right\} \\ \\ \\ \end{array}$$

The merger of previous two clusters into the industrial cluster of dimension $q = 4 : \{MNF, TTF\}$ has a form:

$$\Lambda^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} \quad \mathit{Cluster} \\ \qquad \downarrow \\ \to \{\ MNF\ \} = \left\{ \begin{array}{|c|c|c|c|c|c|} \hline RES & CNS & MNF & TTF & SRV & \quad \\ \hline \end{array} \right\} \\ \to \{\ TTF\ \} = \left\{ \begin{array}{|c|c|c|c|c|c|} \hline RES & CNS & MNF & TTF & SRV & GOV \\ \hline \end{array} \right\} \\ \\ \\ \end{array}$$

This cluster appears as main cluster in the next cumulative incidence matrix (see equation 8). Its structure has the highest dimension $q = 5 : \{MNF, TTF\}$:

$$\Lambda^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} \quad \mathit{Cluster} \\ \qquad \downarrow \\ \to \{\ MNF\ \} = \left\{ \begin{array}{|c|c|c|c|c|c|} \hline RES & CNS & MNF & TTF & SRV & \quad \\ \hline \end{array} \right\} \\ \to \{\ TTF\ \} = \left\{ \begin{array}{|c|c|c|c|c|c|} \hline RES & CNS & MNF & TTF & SRV & GOV \\ \hline \end{array} \right\} \\ \\ \\ \end{array}$$

The augmentation of this cluster has a following form:

$$\Lambda^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} \textit{Cluster} \\ \quad\downarrow \\ \to \{\ MNF\ \} = \{\boxed{RES \mid CNS \mid MNF \mid TTF \mid SRV \mid \quad}\} \\ \to \{\ TTF\ \} = \{\boxed{RES \mid CNS \mid MNF \mid TTF \mid SRV \mid GOV}\} \\ \to \{\ SRV\ \} = \{\boxed{\quad \mid CNS \mid MNF \mid TTF \mid SRV \mid \quad}\} \end{array}$$

The final augmentation of industrial clusters has the following form

$q = 5 : \{MNF, TTF, SRV\}$ (see equation 9) and:

$$\Lambda^4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} \textit{Cluster} \\ \quad\downarrow \\ \to \{\ MNF\ \} = \{\boxed{RES \mid CNS \mid MNF \mid TTF \mid SRV \mid GOV}\} \\ \to \{\ TTF\ \} = \{\boxed{RES \mid CNS \mid MNF \mid TTF \mid SRV \mid GOV}\} \\ \to \{\ SRV\ \} = \{\boxed{RES \mid CNS \mid MNF \mid TTF \mid SRV \mid GOV}\} \end{array}$$

The consecutive stages of the augmentation of industrial clusters can be presented in the form of deepening of complexity (complication) of the sectors of Chicago economy. These results imply the following complication diagram of backward Chenery-Watanabe linkages in Chicago economy in 2000:

31.5%	MNF
57.6%	MNF, TTF
73.3%	MNF, TTF, SRV
83.6%	MNF, TTF, SRV, CNS

This complication diagram of backward linkages (on the aggregation level of six economic sectors) is characteristic for Chicago economy during the all periods, 1975-2000 (see Guo *et al*, 2005).

The analogous Q-analysis of forward linkage sector structure is presented in table 5. In the first two decomposed levels, the number of sectors obtaining the inputs from MNF have decreased since 1980; in 1980, all the six sectors obtain MNF's input (q=5), but by 1985, the number decreases to four (q=4), and since 1990, only four sectors obtain MNF's input inside the Chicago regional economy. On the other hand, the service sector (SRV) has been sending inputs to more and more sectors, increasing from only one sector in 1980 to two in 1985 and three after 1990.

<<insert table 5 here>>

In analogous fashion, the complication diagram for the Chenery-Watanabe backward linkages for 2000 has a form:

$$\begin{array}{ll} 33.7\% & TTF \\ 62.1\% & TTF, MNF \\ 81.2\% & TTF, MNF, SRV \\ 87.1\% & TTF, MNF, SRV, CNS \end{array}$$

This complication diagram is also characteristic for the complication of backward linkages during the period 1980-2000. It is important to note the main key sector for forward linkages is MNF, which is replaced by TTF for backward linkages.

V. Conclusion

While earlier analysis of the production structure in Chicago's economy suggested that the economy was becoming simpler (see Hewings *et al*. 1998) in the sense that the degree of intra-regional intermediation was declining, the analysis of sectoral structure explored a more detailed picture of the changes, showing the relationships of the sectoral structure in different levels of the transaction flows. For about 50 per cent of the total transaction flows, the manufacturing and service sectors have the most noticeable changing features in that manufacturing has less and less connections with other sectors, while the service sectors, on the other hand, expanded their connections with other sectors inside the economy, further indicating their growing importance in the region.

The analysis reveals some features of the structure of Chicago's economy in the last two decades that can be summarized as follows. The results indicate that the production process in Chicago is increasingly becoming more dependent in a backward and forward sense on regions outside the Chicago economy. This result is especially true for manufacturing; the fragmentation of production has been facilitated by the fast growth of the service sectors, especially transportation and communications, that have made it possible to source inputs from distant sources and to serve markets that are more geographically diverse. This kind of production process has been observed internationally (see Jones and Kierzkowski, 1990, 2001a, 2001b). Even though fragmentation of production may happen domestically and internationally, the process has not been documented at the regional level.

The process of cluster development and its evolution remain a challenge; fragmentation may essentially result in a de-clustering process whereby major parts of the value chain of production may be spatially scattered rather than geographically concentrated. Q-analysis offers a methodology that provides a simple way to explore these structural changes. Obviously, the aggregated sectors employed here do not reveal the richness that can be explored; further, with interregional tables, the possible exists to evaluate the way in which structural changes have manifested themselves in the exchange of intra- for inter-regional interactions, thereby generating complication chains that extend far beyond the bounds of traditionally conceived geographically concentrated clusters.

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Table 1 Sector Definitions in the Chicago Input-Output Table

Sector	Name	Content
1	RES	Resources
2	CNS	Construction
3	MNF	Non-durable and durable Goods
4	TTF	Transportation, Trade, FIRE
5	SRV	Services
6	GOV	Government

Table 2. Total Flows for the Chicago Region, 2000

	RES	CNS	MNF	TTF	SRV	GOV	**Total**
RES	29	25	640	183	76	6	**959**
CNS	277	816	2892	5096	4152	574	**13806**
MNF	453	4668	17534	9338	13640	474	**46106**
TTF	431	2578	10697	12041	9284	475	**35506**
SRV	210	2255	6452	8153	6860	229	**24159**
GOV	15	49	566	617	344	41	**1633**
Total	**1416**	**10391**	**38781**	**35428**	**34356**	**1800**	**122171**

Note: RES (resources), CNS (construction), MNF (manufacturing), TTF (trade, transportation), SRV (services), GOV (government)

Table 3 The Coefficient Table and Largest Entries (by column)

	1	2	3	4	5	6
1	0.020662	0.002402	0.016503	0.005169	0.002202	0.00358
2	0.195747	0.078485	0.074577	0.143844	0.120843	0.318719
3	0.319912	0.449218	0.452127	0.263576	0.397018	0.263391
4	0.304647	0.248104	0.275841	0.339856	0.270234	0.263987
5	0.14828	0.217044	0.166361	0.230131	0.19968	0.12735
6	0.010752	0.004746	0.014591	0.017424	0.010023	0.022973

Table 4. The augmentation of the simplices of backward linkages for each industry

$S_0^b(RES)$	$S_1^b(RES)$	$S_2^b(RES)$	$S_3^b(RES)$	$S_0^b(CNS)$	$S_1^b(CNS)$	$S_2^b(CNS)$	$S_2^b(CNS)$
		CNS	*CNS*				
MNF	*MNF*	*MNF*	*MNF*	*MNF*	*MNF*	*MNF*	*MNF*
	TTF	*TTF*	*TTF*		*TTF*	*TTF*	*TTF*
			SRV			*SRV*	*SRV*

$S_0^b(MNF)$	$S_1^b(MNF)$	$S_2^b(MNF)$	$S_2^b(MNF)$	$S_0^b(TTF)$	$S_1^b(TTF)$	$S_2^b(TTF)$	$S_3^b(TTF)$
							CNS
MNF	*MNF*	*MNF*	*MNF*		*MNF*	*MNF*	*MNF*
	TTF	*TTF*	*TTF*	*TTF*	*TTF*	*TTF*	*TTF*
		SRV	*SRV*			SRV	*SRV*

$S_0^b(SRV)$	$S_1^b(SRV)$	$S_2^b(SRV)$	$S_3^b(SRV)$	$S_0^b(GOV)$	$S_1^b(GOV)$	$S_1^b(GOV)$	$S_3^b(GOV)$
		CNS	*CNS*	*CNS*	*CNS*	*CNS*	*CNS*
MNF	*MNF*	*MNF*	*MNF*				*MNF*
	TTF	*TTF*	*TTF*		*TTF*	*TTF*	*TTF*
			SRV				*SRV*

Table 5 Forward linkages sectoral structure

	1980	1985	1990	1995	2000
q=3	{TTF}	{TTF}	{TTF}	{TTF}	{TTF}
q=2	{TTF}	{TTF}	{TTF}	{TTF}	{TTF}
q=1	{TTF} {MNF}	{TTF} {MNF}	{TTF} {MNF}	{TTF} {MNF}	{TTF} {MNF}
q=0	{TTF} {MNF}	{TTF} {MNF}	{TTF} {MNF}	{TTF} {MNF}	{TTF} {MNF}
CP1	0.329	0.332	0.332	0.335	0.337
q=5	{MNF}				
q=4	{MNF}	{MNF}			
q=3	{MNF, TTF}	{MNF}, {TTF}	{MNF}, {TTF}	{MNF}, {TTF}	{MNF}, {TTF}
q=2	{MNF, TTF}	{MNF, TTF}	{MNF}, {TTF}, {SRV}	{MNF}, {TTF}, {SRV}	{MNF}, {TTF}, {SRV}
q=1	{MNF, TTF}	{MNF, TTF}, {SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}
q=0	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF}, {SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}
CP2	0.587	0.607	0.615	0.619	0.621
q=5	{MNF, TTF}	{MNF, TTF}	{MNF, TTF}	{MNF, TTF}	{MNF, TTF}
q=4	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}
q=3	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}
q=2	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}
q=1	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}
q=0	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}
CP3	0.770	0.794	0.801	0.808	0.812
q=5	{MNF, TTF}	{MNF, TTF}	{MNF, TTF}	{MNF, TTF, SRV}	{MNF, TTF, SRV}
q=4	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}
q=3	{MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}	{MNF, TTF, SRV}	{MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}
q=2	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}
q=1	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}
q=0	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}	{CNS, MNF, TTF, SRV}
CP4	0.836	0.849	0.858	0.866	0.871

λ	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
S1	1	0	1	0	0	1	0	1	1	1
S2	1	1	0	0	0	0	1	1	0	0
S3	0	1	1	1	1	0	0	1	0	1
S4	1	0	1	0	0	0	0	0	0	0
S5	0	1	0	0	0	1	0	0	1	1
S6	0	1	0	0	0	0	0	1	0	0
S7	0	0	1	0	0	1	0	0	0	0
S8	0	1	0	0	1	0	0	1	0	1
S9	1	0	1	0	0	1	0	0	0	0
S10	0	1	1	0	0	0	1	1	1	0

Shared faces are indicated by cells with a dotted box

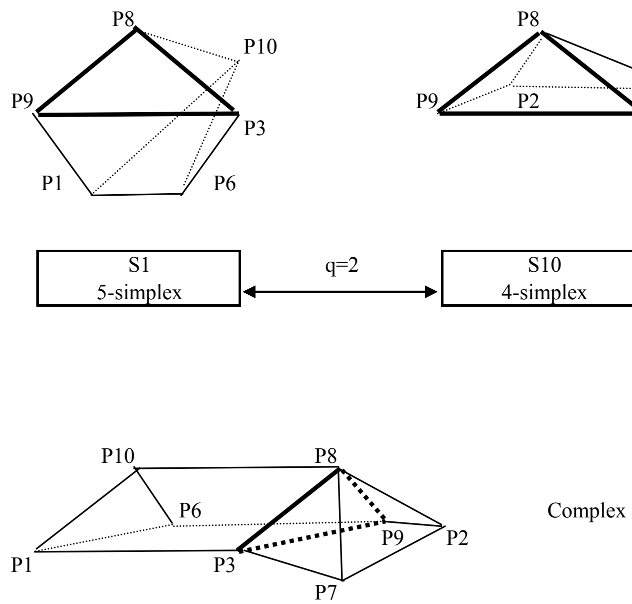

Figure 1: Structure of Relationships: Simplices, Shared Faces and a Complex