ENDOGENOUS GROWTH OF AGEING ECONOMY: EVIDENCE AND POLICY MEASURES

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ABSTRACT: This paper adopts the two-sector overlapping generation (OLG) model to capture the impact of population ageing on the regional economy and compares the effectiveness of government policy in an endogenous growth perspective. Comparing the computational results of a one-sector OLG model, where agent's productivity is given exogenously, this paper confirms that endogenously determined investment in human capital significantly offsets the negative effects of the ageing population on the regional economy. This paper also attempts to check if there is room for the government to weaken and prevent the negative effects of the ageing population. For this, this paper examines the effects of two kinds of government transfer systems on the regional economy: money transfer and educational transfer systems. The money transfer, which is redistributed to agents by the government, could be used for an individual's consumption, saving and educational investment. Educational transfer is given directly to the individual proportional to his or her opportunity cost stemming from education investment. The result shows that the educational transfer system is superior to money transfer system in the long-run in terms of growth of per-capita income, aggregate welfare and stabilizing the factor prices. However, the result implies that there exists a trade-off relationship in implementing an educational transfer system between economic growth and equity of income and wealth.

KEY WORDS: Endogenous Growth; Human Capital; Overlapping Generations; Population Ageing; Tax and Transfer

1 Introduction

The impact of an aging population on economy is one of the key issues in most developed countries. In particular, the dependency ratio[1] of the US is expected to rise rapidly until mid 2030's. Table 1 shows that the ratio in the US is projected to increase from 19% in 2004 to 33% in 2035 and then level off at around 33%. The growth of dependency ratio is projected to peak in 2025 and then slow down (figure 1). This change of trend reflects the impact of "baby boomers" born in the post-World War II period from 1946 through 1964 who will be approaching retirement age sequentially over the next two decades; the oldest baby boomers turn 60 in late 2000s and the youngest turn 60 in late 2020s. The state of Illinois[2] also shows similar trends except that the dependency ratio of Illinois is anticipated to be less than national ratio (see table 1).

An aging population could cause negative effects on the economy, including a decrease of per-capita output and economic welfare, mainly due to the decline in the supply of the labor force and the saving rate: the dissaving of the old should increasingly offsets the saving of the young during the ageing era. Therefore, it becomes important to forecast the impacts of an aging population on the economy and to assess the options for government policies that might be needed to address the challenge.

A two-sector overlapping generation (OLG) model is developed to capture the impact of population ageing on Illinois and compare the effectiveness of government policy in an endogenous growth perspective. The endogenous growth model has attempted to endogenize the underlying source of sustained growth. In the case of a two-

[1] Dependency ratio = number of population over 65 / number of population between 15 and 64.
[2] U. S Census Bureau projected population's age structure of each state up to 2030 and national population's age structure up to 2050.

sector endogenous growth model, production of human capital is supposed to be relatively intensive in human capital. Consequently, each individual's education activity plays a key role in economic growth. For example, in the Uzawa (1965) model, there is no physical capital involved in production of human capital while in the Lucas (1988) model, the total factor productivity in the final good sector depends on the average level of human capital in an economy.

Sadahiro and Shimasawa (2002) developed the overlapping generation model (OLG) based on two-sector endogenous growth theory in order to project the effects of population ageing impact on the economy. They endogenized the individual's choice in human capital investment at the workplace; and found out that endogenously determined investment in human capital offsets significantly the negative effects of ageing population. However, they did not apply their model to an actual population structure; and also paid little attention to the role of government policy. Ludwig *et al.* (2007) extended Sadahiro and Shimasawa (2002) with different formations of human capital accumulation.

This paper is organized as follows. In section II, the model description and the numerical algorithm will be demonstrated. In section III, the computational results of the baseline model are presented, compared to those with a one-sector OLG model, which does not take endogenous growth of human capital into consideration (this kind of model will be referred to as an "exogenous model" later). In section IV, the government's policy to address the negative effects of ageing population on the regional economy will be simulated. In the final section, the conclusion will be drawn and the further research subject will be briefly discussed.

2 Baseline Model

There are three types of agents in the baseline model: representative households, firms and government. The households maximize utility, subject to the usual budget constraint. Firms hire labor and rent physical capital to produce physical goods in the competitive market. Government levies pension tax on the workers and operate the social pension system of "pay-as-you-go" type with the tax revenue. A two-sector economy is proposed with physical goods and human capital sectors. The target period is 2001 through 2050 when the ageing phenomenon is projected to deepen in Illinois as well as the U. S. It is assumed that there is no uncertainty in the economy. There exist J generations in every single year and the generations are overlapped every sample period.[3]

2.1 Households

At the beginning of age 1[4], each individual makes a decision on allocating resources on consumption and savings as well as splitting the endowment time into schooling and working for a whole life-time to maximize his/her life-time welfare. It is assumed that the individual enters into the labor market at age 1 and retire at age j^*. Every agent is supposed to live until age J. The instantaneous utility function has two arguments, consumption and investment in human capital:

[3] Thus, we could call this paper's model as perfect foresight OLG model.
[4] Note that age 1 in the model corresponds to age 20 in reality.

$$u(c_{t,j},e_{t,j})=\frac{c_{t,j}^{\ 1-\gamma}+\theta e_{t,j}^{\ 1-\gamma}}{1-\gamma}\quad \gamma>1,\ 0<\theta<1 \tag{1}$$

where $c_{t,j}$ is consumption and $e_{t,j}$ is time fraction of investment in human capital[5] at time t and age j while θ is the parameter of the degree of educational investment motive and γ is the parameter of the inverse of the inter-temporal elasticity of substitution.

Therefore, the individual's life-time utility function is as follows:

$$U_t^{\ 1}=\sum_{j=1}^{J}\beta^{j-1}u(c_{t+j-1,j},e_{t+j-1,j})=\sum_{j=1}^{J}\beta^{j-1}\left(\frac{c_{t+j-1,j}^{\ \ 1-\gamma}+\theta e_{t+j-1,j}^{\ \ 1-\gamma}}{1-\gamma}\right) \tag{2}$$

$U_t^{\ 1}$ denotes the life-time utility of the individual who is born in the year t and β denotes the parameter of subjective discount rate. The individual who was born in time t has a following life-time budget constraint:

$$\sum_{j=1}^{J}\left(\left(\prod_{k=t}^{t+j-2}\frac{1}{1+r_k}\right)c_{t+j-1,j}\right)=\sum_{j=1}^{j^*}\left(\left(\prod_{k=t}^{t+j-2}\frac{1}{1+r_k}\right)(1-\tau_{t+j-1}^{p})h_{t+j-1,j}w_{t+j-1}(1-e_{t+j-1,j})\right)+$$

$$+\sum_{j=j^*+1}^{J}\left(\left(\prod_{k=t}^{t+j-2}\frac{1}{1+r_k}\right)pen_{t+j-1,j}\right) \tag{3}$$

where r_t is the real interest rate , w_t is the wage rate and τ_t^p is the social security tax rate at time t while $h_{t,j}$ is the human capital stock and $pen_{t,j}$ is the level of pension benefit at time t and age j. Every new generation in each year maximizes the life-time utility function (2) under the budget constraint (3). The Euler equations (4) and (5) could be

[5] In the model, the individual whose age is between 1 and j^* allocates his/her endowment time (=1) into labor and education investment. Therefore, $0\le e_{t,j}\le 1$ for $j=1,....,j^*$.

derived by computing the first order conditions with regard to consumption, saving and education investment time:

$$c_{t+1,j+1} = \left(\beta\left(1+r_{t+1}\right)\right)^{1/\gamma} c_{t,j} \tag{4}$$

$$e_{t,j} = \left(\frac{\theta}{(1-\tau_t^p)w_t h_{t,j}}\right)^{1/\gamma} c_{t,j} \text{ if } j \le j^*. \tag{5}$$ [6]

Note that, in this model, spending time in educational investment is supposed to affect agent's utility in three channels: (1) educational investment time ↑ → utility ↑ (short-run direct channel), (2) educational investment time ↑ → labor time ↓ → labor income ↓ → consumption ↓ → utility ↓ (short-run indirect channel) and (3) educational investment time ↑ → human capital stock ↑ → labor productivity ↑ → labor income ↑ → consumption ↑ → utility ↑ (long-run indirect channel). As for the 3rd channel, Heckman *et. al* (1998) revealed that post-school learning, including job search, learning-by-doing and workplace education, accounts for 1/3 to 1/2 of all skill formation in a modern economy.

An individual's wealth, which comprises accumulated personal saving over time, at time t and age j (=$a_{t,j}$) evolves as follows:

$$a_{t+1,j+1} = (1-\tau_t^p)h_{t,j}w_t(1-e_{t,j}) + (1+r_t)a_{t,j} - c_{t,j} \text{ if } 1 \le j \le j^* \tag{6}$$

$$a_{t+1,j+1} = pen_{t,j} + (1+r_t)a_{t,j} - c_{t,j} \text{ if } j > j^*.$$

Aggregate supply of physical capital stock at time t is:

[6] Note that we have a boundary condition $e_{t,j} \le 1$.

$$K_t^s = \sum_{j=1}^{J} a_{t,j} N_{t,j} \tag{7}$$

where $N_{t,j}$ denotes population size of age-cohort j in time t. The aggregate consumption at time t is:

$$C_t = \sum_{j=1}^{J} c_{t,j} N_{t,j} \,. \tag{8}$$

2.2 Firm

The firm produces a composite good by renting physical capital and hiring labor in order to maximize its profit each year. The Cobb-Douglas production function is used with the following specification:

$$Y_t = A(K_t^d)^{\alpha}(L_t^{e\ d})^{1-\alpha} \tag{9}$$

where K_t^d the demand of physical capital and $L_t^{e\ d}$ is the demand of effective labor at time t while A is the parameter of total factor productivity and α is the parameter of physical capital income share. Factor prices are determined in the competitive market:

$$r_t = \alpha A(K_t^d)^{\alpha-1}(L_t^{e\ d})^{1-\alpha} - \delta \tag{10}$$

and $w_t = (1-\alpha)A(K_t^d)^{\alpha}(L_t^{e\ d})^{-\alpha}$ (11)

where δ is physical capital depreciation rate.

2.3 Government

The government in the baseline economy just operates the social security system; the government levies a social security tax on labor income and transfers the pension benefit to retirees. The government's budget is assumed to be balanced every period:

$$\tau_t^p \left(\sum_{j=1}^{j^*} N_{t,j} \left((1-e_{t,j}) w_t h_{t,j} \right) \right) = \sum_{j=j^*+1}^{J} N_{t,j}\, pen_{t,j} \,. \tag{12}$$

The magnitude of annual pension benefit of a retiree is dependent on his/her average yearly (gross) labor income before retirement. The government transfers a pension benefit to a retiree which amounts to his/her yearly average labor income multiplied by replacement ratio (ξ).

2.4 Human capital

Generally an individual's human capital stock is embodied privately so it has a property of rivalry and exclusiveness**;** the use of embodied skills in one activity precludes their use in another activities ("rivalry") and people have property rights in their own skills ("exclusiveness"). Therefore, to maintain and improve aggregate human capital stock in the economy, there should be a transferring mechanism of human capital stock from generation to generation; this transferring mechanism is referred to as "education." Following the human capital production function of Sadahiro and Shimasawa (2002), the process may be presented as:

$$h_{j+1,t+1} = (1-\delta_h) h_{j,t} + B(mk_t)^{\phi} (h_{j,t} e_{j,t})^{1-\phi} \tag{13}$$

where k_t is the physical capital/labor ratio while B is the parameter for accumulation efficiency of human capital, m is the portion of physical capital stock for producing the human capital stock, δ_h is the parameter of depreciation rate of human capital stock and ϕ is the parameter of elasticity of human capital formation function. This human capital production function includes the physical capital ratio and the degree of accumulation efficiency of human capital stock so that the model could reflect the efficiency of education system of the corresponding economy. Note that there is a need to develop a rule of assigning a human capital stock of age 1 individuals of each year. The proposal of Sadahiro and Shimasawa (2002) is adopted so that the new generation is born with a portion of human capital stock of the previous generations:

$$h_{t,1} = \pi^{hc}\left(\left(\sum_{j=1}^{j*} h_{t-1,j} N_{t-1,j}\right) / \sum_{j=1}^{j*} N_{t-1,j}\right) \tag{14}$$

where π^{hc} is the parameter of efficiency of human capital transmission between generations. Aggregate human capital stock at time t is defined as:

$$H_t = \sum_{j=1}^{j*} h_{t,j} N_{t,j} \ . \tag{15}$$

Then, aggregate supply of effective labor can be computed as shown in (16):

$$L_t^{e\ s} = \sum_{j=1}^{j*} (1 - e_{t,j}) h_{t,j} N_{t,j} \ . \tag{16}$$

2.5 Market clearing

Factor and goods market clearing conditions hold at every period as follows:

$$L_t^{e\ s} = L_t^{e\ d}\left(\equiv L_t^e\right) \tag{17}$$

$$K_t^s = K_t^d\left(\equiv K_t\right) \tag{18}$$

$$Y_t = C_t + I_t + G_t$$ (19)[7]

where $I_t = K_t - (1-\delta)K_{t-1}$.

2.6 Calibration

Parameters

This paper adopts the relevant parameters from the related previous literature (see table 2). In particular, the parameters associated with the human capital production are from Sadahiro and Shimasawa (2002). The other parameters are mainly from Park and Hewings (2007). Also note that it is for the period 2001 through 2050 that simulation results are presented in the next section.

Age-cohort structure

As for the age-cohort distribution, calibrated to our model, the U. S. Census Bureau (USCB)'s projections that are available up to 2030 are used. The USCB's projection are extended to 2050, assuming that the correlation of age-cohort structures between Illinois and the U. S will be equivalent as for the previous period. Figure 2 shows the changes of age-cohort structures of Illinois from 2010 thru 2030 to 2050. The impacts of this demographic change will be explored in the next section.

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[7] In this model, G_t=0.

Initial distribution

One of the difficulties the paper should address is how to set the initial distribution of wealth and human capital stock in 2001, which is the beginning year of economy of our model. The economy of Illinois was not at a steady state in 2001; hence, this paper shows the another way to address this issue, instead of adopting a steady state distribution of wealth and human capital stock as a distribution in 2001.

First, for the distribution of human capital stock in 2001, the productivity age profile[8] adopted by Park and Hewings (2007) was used so that it would be possible to compare the simulation results between the one-sector model ("exogenous model") and the two-sector model ("endogenous model"), that have different views on human capital and its role on economic growth.

Secondly, the initial wealth distribution might be calibrated empirically. However, the impacts of factors[9] other than ageing population and government's fiscal policies should be excluded in projecting the economic variables. Thus, the initial wealth distribution in an endogenous approach can be generated like the following. It was assumed that the cross-sectional wealth distribution in 2001 was exactly the same as the life-cycle wealth profile of age 1 generation in 2001. Starting with the arbitrary initial wealth distribution then the life-cycle wealth profile of each generation in each period

[8] Park and Hewings (2007) assumed an individual's labor productivity to be an exogenous function of his/her age: $e_j = \lambda_1 + \lambda_2 j - \lambda_3 j^2$ and calibrated the parameters: $\lambda_1, \lambda_2, \lambda_3$.

[9] For example, uncertainties surrounding the economy affect the saving motive of the consumers remarkably in reality. We do not want these uncertainties, which exist outside the model, affect our simulation results.

can be calculated. Our interest is in whether the wealth distribution across generations in 2001 is identical to the life-cycle profile of age 1 generation in 2001. The fixed-point iteration method is used to get the convergence of wealth distribution.

Wealth distribution across generations in 2001 that is developed by the method mentioned above has a usual life-cycle pattern: wealth is accumulated until the retirement age and then decreases close to zero since there is no bequest motive in consumer's utility function in the model (figure 3). Again, note that, by assumption and for comparison purposes, the initial wealth distribution in 2001 is identical in both of models: the exogenous productivity model and the endogenous human capital model.

3 Computational Results

In this section, the simulation result of baseline model will be presented. The results from the endogenous human capital model will be compared to the simulation results of the exogenous model in order to derive the implications of the roles of human capital during the era of an ageing population. As mentioned before, the exogenous productivity model assumes that the individuals devote his/her whole endowment time into working. On the contrary, the individuals in the other model have an incentive of splitting his/her endowment time into working and human capital investment.

3.1 Per-capita output

During our sample period, i.e., 2001 through 2050, per-capita output continues to grow under the endogenous human capital model. In contrast, under the exogenous productivity model, per-capita output increases for the early stage but begins to decrease in the early 2010s and then levels off in the late 2030s (see figure 4). Therefore, the per-capita output of endogenous human capital model is 24% larger than that of the exogenous productivity model in 2050. Note that the per-capita output of the exogenous case is larger than that of the endogenous case for the early years of our sample period since the individuals do not devote their whole endowment time into working. However, the increasing productivity of the endogenous case dominates the relative deficiency of labor contributions at an early stage.

Figure 2 showed that the impact of the ageing phenomenon is forecast to be most profound during the 2010s and 2020s. According to the simulation results of the endogenous human capital model, per-capita output grows 0.44 and 0.12 percent on average during the 2010s and 2020s and then recovers relatively quickly by the 2030s. As table 3 shows, the improvement of total productivity dominates the negative effect of decreases in the labor force in the case of endogenous human capital model. The scarcity of labor decreases the growth of per-capita output by 0.27 percent point and 0.29 percent point during 2010s and 2020s respectively, while increasing total productivity accelerates the output growth by 0.47 percent point and 0.37 percent respectively. On the contrary, the exogenous productivity model forecasts that the economy will suffer the negative growth of per-capita output during 2010s and 2020s and then recover slowly from the impact of an ageing population.

3.2 Factor prices

The exogenous productivity model forecasts that factor prices such as interest and wage rates will go through considerable change during the phase of ageing population. In the ageing economy, the labor (physical capital) becomes relatively scarce (abundant), therefore the factor price of labor (physical capital) moves higher (lower). However, according to the endogenous human capital model, the agents determine his/her life-cycle profile of working and time shares of human capital investment, based on his/her expectation about future path of each factor prices. The implications are that the agents in the endogenous model tend to increase his/her time share of investment in his/her human capital since he/she knows the wage rate (interest rate) will become higher (lower) in the future and his/her human capital stock will grow proportionally to his/her investment time.[10] This kind of mechanism, in the endogenous human capital model, makes the factor prices to show relatively stable movements during the ageing period, compared to those of the exogenous productivity model. Figure 5 shows the simulation results confirming these conjectures.

Since ageing does not cause a considerable drop in the interest rate or a rise in the wage rate, the aggregate saving rate[11] does not decline sharply during the ageing period in spite of the same projection of the demographic transition (figure 6). The endogenous model implies that the aggregate saving rate will drop by 6.6 percentage points from 2007 to 2031. On the contrary, the exogenous model implies that it will decline by 11.7 percentage points during the same period.

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[10] See (13).

[11] Saving rate (%) = aggregate saving / aggregate disposable income.

3.3 Welfare

In this section, the simulation results for social welfare will be presented. Figure 7 shows the consumption level corresponding to aggregate social welfare.[12] Social welfare grows gradually in the case of the endogenous model, but it seems to stagnate in the case of the exogenous model. However, it takes a long time for the social welfare of endogenous model to catch up with the social welfare of the exogenous model.

This result may be ascribed to the fact that the wage rate of the endogenous model does not surge as in the case of the exogenous model. On the other hand, a relatively high interest rate in the endogenous model cannot cancel the negative effect of low wage rates since young agents enter the labor market without any asset holdings. Also, note that it takes some time to build up the human capital stock to the level that could dominate the low wage rate effect. Thus, the ratio of average welfare of working cohorts to the retired cohorts in the endogenous model is relatively low, compared to that of exogenous model (table 4).

3.4 Sensitivity analysis

Majority of the parameters are accepted from the previous relevant literature. In this section, the result of sensitivity analysis associated with the key parameter values will be presented. One parameter which will be taken into consideration is the elasticity of inter-temporal substitution (EIS: $1/\gamma$), which measures the extent to which household agents

[12] Due to the utility functional format, welfare is computed to be a negative value. Note that magnitude of utility does not matter: that is utility should be considered as ordinal, not as cardinal.

substitute their consumption inter-temporarily in response to the expected changes in the real interest rate in the future. The other parameters of interest are the efficiency of human capital formation (B) and retirement age ($j*$).

The results show that the per-capita output and its growth are substantially influenced with the magnitude of γ and B. First, the left panel in figure 8 shows that the lower EIS (case of γ=2.6) increases the growth rate of per-capita output during the ageing era. However, the effect of lowering EIS attenuates gradually along the years. On the contrary, under the higher EIS (case of γ=1.2), the growth rate was negative most of years during 2001 through 2050. This is mainly because the tendency to defer less consumption to the future, which was triggered by the expectation that interest rate declines during the ageing era, is promoted by increasing the EIS. This in turn induce the individuals to put less weight on the schooling since the schooling just help the individuals to consume more in the future and make consume less in the current period; thus the average productivity of the workers (human capital stock) falls, compared to the model economy with the lower EIS (right panel in the figure 8).

Second, the parameter (B) connected directly to the efficiency of human capital formation significantly affects the performance of the model economy. When parameter B is set to be .38, which is .10 points higher than baseline, the growth rate of per-capita output becomes substantially higher (figure 9). This matches the intuition quite well since the individuals put more weight on their schooling when the return to education is higher when every thing is held constant. However, the whole dynamic pattern of growth of per-capita output are similar each other regardless of magnitude of this parameter value.

Third, as the individuals expect to live longer, they plan to extend their retirement age since they need more income to support their lengthened life time unless they rationally expect a favorable change in the public pension system. Also in a point view of government, extension of retirement age seem to be desirable since the prime source of negative effects of population ageing is the deficiency of labor force.

The simulation results reveal that the effects of extending the retirement age cannot be ignored in terms of boosting the per-capita output: the per-capita output was increased about .5 percent from the baseline when extending one more year of retirement age (left panel in figure 10). However the effects are not enlarged along the years; thus the growth rate of per-capita output was not affected by extending the retirement age (right panel in the figure 10). Furthermore, the growth rate seems to get the downward pressure in the long term by extending the retirement age.

The main reason why the impacts on the economic performance are not accumulated inter-temporarily could be traced to the way how the human capital forms in the model (see (13)). If the retirement age is extended, the young should share the educational system, which is the constant portion of physical capital stock according to the model specification[13], with the old who were not supposed to have participated in the education system unless the retirement age was extended; and more condensed educational environment lowers the efficiency in building up the human capital stock of

[13] In our model, the physical capital which is used for the education is denoted by mk_t in (13).

younger workers. Furthermore, the elderly who belong to the age-cohort over than previous retirement age are less efficient in developing their productivity; and they will retire soon. Therefore, the average labor productivity in the economy gets the negative pressure by extending the retirement age. This negative pressure on the average productivity is accumulated generation to generation through the human capital transmission mechanism. In case of extending 5 years of retirement age, the average productivity declines by .86 percent in the initial year (2001) and decrease by 3.42 percent in 2050 (table 5).

4 Government policies

This section tries to answer the following question: "Which government policy is effective to mitigate the effect of ageing population?" In the last section, we have seen that simply extending the retirement age does not guarantee the sustainable growth during the ageing era. One option for effective government policy should be to alter individual's choices so that the equilibrium outcomes should be ones that sustain the higher economic growth and social welfare with the same amount of budget.

In this chapter, the focus is restricted to the tax and transfer policy, more specifically, money transfer and educational transfer. For reference, Seshadri and Yuki (2003) explored the effects of a government's redistributive policy on the equity and efficiency in the model economy. Using the simple two-period stochastic OLG model, they showed that the educational transfer enhances the efficiency and reduces the inequality in the steady state equilibrium.

Money transfer is a kind of redistribution of wealth. In the simulation used here, a lump-sum money transfer is given to each household who is in the working age-cohort regardless of its income, productivity and wealth levels. It could be used as a source for consumption, saving and education investment.[14]

Educational transfer is given to an individual proportionally to his/her opportunity cost of education investment. In other words, the primary purpose of educational transfer policy is to encourage the agents to invest in their human capital. The background of this policy should be that the average workers who are older than before (thanks to the population ageing) should be less active in increasing their investment in their human capital due to the low marginal return to schooling; on the contrary the worker's skill at the workplace are rapidly made obsolete under the population ageing. Grip and Loo (2001) classifies skill obsolescence into two categories according to its cause: "technical" and "economic" skill obsolescence. The cause of "technical" obsolescence is found at the natural wear of skills; while "economic" skill obsolescence is attributed to the external causes such as technological developments and shifts of the demand for skill levels from the industries. First, it should be obvious that population ageing may cause rapid "technical" skill obsolescence in the aggregate level. Also, there are increasing empirical studies showing that the demand for the low skilled is rapidly declining mainly due to the fast technical progress (Goldin and Katz, 1998 and Bresnahan *el al.,* 2002). Thus, the purpose of educational transfer should be understood as help to decelerate the

[14] In our endogenous model, education expenditure is not a direct argument in household's budget constraint, but education time is an argument of utility function. Note that increasing consumption, thanks to money transfer, could weaken the incentive of laboring so that agent will increase his/her education time fraction.

speed of obsolescence of the aggregate skill level in the corresponding economy, caused by population ageing, as well as to attach the low-skilled persons to the fast changing industrial skill demand.

Now, the setup of each sector under each scheme of government policy will be presented. Note that firm and human capital sectors are identical to our baseline model and market clearing conditions are straightforward enough to be omitted in this section.

4.1 Households

Educational transfer

When the government implements an educational transfer system, an agent who was born in year t has a following inter-temporal budget constraint:

$$\sum_{j=1}^{J}\left(\left(\prod_{k=t}^{t+j-2}\frac{1}{1+r_k}\right)c_{t+j-1,j}\right)=\sum_{j=j^*+1}^{J}\left(\left(\prod_{k=t}^{t+j-2}\frac{1}{(1-\tau^e_{t+j-1})r_k}\right)pen_{t+j-1,j}\right)+$$

$$\sum_{j=1}^{j^*}\left(\left(\prod_{k=t}^{t+j-2}\frac{1}{1+(1-\tau^e_{t+j-1})r_k}\right)h_{t+j-1,j}w_{t+j-1}\left((1-\tau^p_{t+j-1}-\tau^e_{t+j-1})(1-e_{t+j-1,j})+\mu e_{t+j-1,j}\right)\right) \tag{20}$$

where μ is the reimbursement rate, which represents how much portion of opportunity cost of an individual's educational investment is covered by the government's educational transfer, and τ^e_t is the education tax rate at time t. The Euler equations (4) and (5) should be modified as follows:

$$c_{t+1,j+1}=\left(\beta\left(1+(1-\tau^e_{t+1})r_{t+1}\right)\right)^{1/\gamma}c_{t,j} \tag{21}$$

$$e_{t,j} = \left(\frac{\theta}{\left((1-\tau_t^p - \tau_t^e) - \mu\right) w_t h_{t,j}} \right)^{1/\gamma} c_{t,j} \text{ if } j \le j^*. \tag{22}$$[15]

An individual's wealth, that is accumulated personal savings over time, at time t and age j $(= a_{t,j})$ evolves by the following equations:

$$a_{t+1,j+1} = h_{t,j} w_t \left((1-\tau_t^p - \tau_t^p)(1-e_{t,j}) + \mu e_{t,j} \right) + (1+(1-\tau_t^e) r_t) a_{t,j} - c_{t,j} \text{ if } 1 \le j \le j^* \tag{23}$$

$$a_{t+1,j+1} = pen_{t,j} + (1+(1-\tau_t^e) r_t) a_{t,j} - c_{t,j} \text{ if } j > j^*. \tag{24}$$

Money transfer

When the government operates the money transfer system, household's inter-temporal budget constraint is as follows:

$$\sum_{j=1}^{J} \left(\left(\prod_{k=t}^{t+j-2} \frac{1}{1+r_k} \right) c_{t+j-1,j} \right) = \sum_{j=j^*+1}^{J} \left(\left(\prod_{k=t}^{t+j-2} \frac{1}{(1-\tau_{t+j-1}^e) r_k} \right) pen_{t+j-1,j} \right) +$$

$$\sum_{j=1}^{j^*} \left(\left(\prod_{k=t}^{t+j-2} \frac{1}{1+(1-\tau_{t+j-1}^e) r_k} \right) \left(h_{t+j-1,j} w_{t+j-1} (1-\tau_{t+j-1}^p - \tau_{t+j-1}^e)(1-e_{t+j-1,j}) + \chi_{t+j-1,j} \right) \right) \tag{25}$$

where $\chi_{t,j}$ is the money transfer from government to individual whose age is j at time t.

The individual's dynamic optimization problem can be solved as:

$$c_{t+1,j+1} = \left(\beta \left(1 + (1-\tau_{t+1}^e) r_{t+1} \right) \right)^{1/\gamma} c_{t,j} \tag{26}$$

$$e_{t,j} = \left(\frac{\theta}{\left(1-\tau_t^p - \tau_t^e\right) w_t h_{t,j}} \right)^{1/\gamma} c_{t,j} \text{ if } j \le j^*. \tag{27}$$

[15] Note that $\tau_t^p + \tau_t^e < 1 - \mu$.

The individual's wealth at time t for workers evolves by the following equation while the wealth for the retired agents evolves by (24):

$$a_{t+1,j+1} = (1-\tau_t^p - \tau_t^e)h_{t,j}w_t(1-e_{t,j}) + (1+(1-\tau_t^e)r_t)a_{t,j} + \chi_{t,j} - c_{t,j} \text{ if } 1 \le j \le j^* \quad (28)$$

4.2 Government

When the government operates the educational transfer system, it levies an educational tax on household's income and reimburses proportionally to his/her opportunity cost stemming from time spent on educational investment. It is assumed that the government's social security system and educational transfer system are operated independently from each other. Also the assumption that government's budget is balanced every period is maintained. Therefore, the budget constraint corresponding to the educational transfer system is as follows while the constraint of social security system is same as (12):

$$\tau_t^e\left(\left(\sum_{j=1}^{j^*} N_{t,j}\left((1-e_{t,j})w_t h_{t,j}\right)\right) + \left(\sum_{j=1}^{J} N_{t,j}\left(r_t a_{t,j}\right)\right)\right) = \mu \sum_{j=1}^{j^*} N_{t,j}\left(e_{t,j} w_t h_{t,j}\right). \quad (29)$$

When the government operates the money transfer system, it also levies a tax on household's income and transfers the lump-sum amount to each worker. Further, the government's social security system and money transfer system are operated independently from each other. The budget constraint of money transfer system is as follows:

$$\tau_t^e\left(\left(\sum_{j=1}^{j^*} N_{t,j}\left((1-e_{t,j})w_t h_{t,j}\right)\right) + \left(\sum_{j=1}^{J} N_{t,j}\left(r_t a_{t,j}\right)\right)\right) = \sum_{j=1}^{j^*} N_{t,j}\chi_{t,j}. \quad (30)$$

Note that the economy is simulated under two different government policy schemes: educational transfer and money transfer systems. However, it is assumed that the budget volumes of these two systems are identical at every period for comparison purpose.

4.3 Computational result

4.3.1 Per-capita output

In the short-run, both of the government policy systems decrease per-capita output since these transfer systems weaken the incentive to work. Figure 11 implies that the short-run negative effect is much larger when the government operates an educational transfer system than when it operates money transfer system. For example, the educational transfer system which reimburses 30% (20%, 10%) of opportunity cost of educational investment decreases per-capita output by 1.99% (1.16%, 0.52%) in 2001, compared to the per-capita output of baseline model in 2001. On the contrary, the money transfer system whose budget volume is equivalent to that of the educational transfer system lowers just the output by 0.21% (0.12%, 0.06%) in 2001.

However, the government's transfer system boosts the economy in a long-run. In particular, the positive effect of educational transfer is noteworthy whereas money transfer system barely increases the per-capita output even though government expends same amount of budget every single year. For example, educational transfer system whose reimbursement rate is 30% (20%, 10%) increases per-capita output by 3.78%

(2.39%, 1.14%) in 2050, compared to the per-capital output of baseline model in 2050. However, the money transfer system increases per-capita output by 0.28% (0.17%, 0.08%) in 2050.

The government's transfer policy increases the incentive of investment in human capital. This increasing incentive causes an improvement in total productivity in the economy and finally boosts the economy during the ageing period. Table 6 shows that the educational transfer encourages individuals to allocate more time to schooling than the money transfer option does even though the money transfer consumes the equivalent government's budget as the educational transfer system every year. The educational transfer system whose reimbursement rate is 30% (20%, 10%) increases the average time share of educational investment by 27.87% (16.04%, 7.10%), compared to the baseline model where no transfer policy is involved. On the other hand, the money transfer system increases the time share of education investment only by 3.08% (1.80%, 0.81%), compared to the baseline model. It should be also noticed that individuals devote more time into investing in his/her human capital as the reimbursement rate is higher or government transfer more lump-sum money amount to the individuals.

4.3.2 Factor prices

In the last section, notice that when the government implements a transfer policy regardless of educational or money transfer, the individuals are encouraged to allocate more time into his/her human capital investment. From this result, it can be assumed that the relative scarcity of labor (or relative abundance of physical capital) during the demographic transition would be mitigated due to further improvement of productivity if the government implements transfer policies. Table 7 confirms these conjectures. The educational transfer system soothes the phenomenon of relative scarcity of labor (or relative abundance of physical capital) more effectively than money transfer system does although it consumes the same budget volume as the educational transfer system. Also, the mitigating effect is larger as the government transfers more.

During 2010 through 2030, when the ageing phenomenon peaks, the rise of the wage rate (decrease of interest rate) resulting from the scarcity of labor force could be weakened if government implements a transfer system to counteract ageing phenomenon (figure 12). However, it should be noted that in an early stage of the government's policy implementation, the economy could receive a temporary labor shortage shock since agents begin to allocate more time into education than before and total productivity is not yet high enough to cover this shortage of labor time.

The educational transfer system is more effective than the money transfer system in mitigating less-saving tendency during ageing periods. Furthermore, the aggregate saving rate could be increased more as government reimburses more under the educational transfer system (figure 13). On the contrary, there is no notable positive effect of the money transfer system regardless of its magnitude. Again, it should be noted that aggregate saving rate could be lower than in the baseline case in an early stage of government's policy implementation because household's saving drops mainly due to decrease of income stemming from the shortage of working hours and the immature status of human capital stock.

4.3.3 Welfare

The educational transfer increases social welfare significantly while money transfer's effect is trivial regardless of the magnitude of transfer. Educational transfers affect social welfare through two channels: education and consumption. Individuals are encouraged to spend more time in schooling and this increase in educational investment positively affects directly the individual's welfare. Furthermore, rising schooling time promotes an individual's human capital stock so that he/she could enjoy more consumption in the future. Since human capital stock has to be accumulated, this positive effect is accelerated as time passes unless government changes its policy schemes. Table 8 shows that a persistent educational transfer system accelerates growth of social welfare. The educational transfer system that reimburses 30% (20%, 10%) of the opportunity cost of human capital investment promotes 2.70% (1.73%, 0.83%) compared to the baseline case during 2000's. During the 2040's, the educational transfer whose reimbursement rate is 30% (20%, 10%) improves social welfare by 6.29% (3.92%, 1.84%).

4.3.4 Economic equity

The simulation results show that the government's transfer policies do not improve equality of income and wealth distribution. Instead, both of the educational transfer and money transfer policies worsen equality of income distribution. Also, the educational transfer policy even deteriorates the equality of wealth distribution. Note that the money transfer improves the equality of wealth distribution but the degree of improvement seems to be very small (figure 14).

It can be seen that the Gini coefficient of income rises more as government increases its reimbursement rate and transfers more money. In the case of the Gini coefficient of wealth, the coefficient becomes higher as the government's reimbursement rate of educational transfer system increases while the money transfer lowers the coefficient a little.

These results imply that the government transfer regime does not work so effectively for the agent with low human capital stock while the policy encourages efficient agents to invest more effectively in his/her human capital stock.

5 Conclusions

The ageing phenomenon is forecast to peak from 2010 to 2030 in case of Illinois; and it has been shown that the population ageing is expected to affect Illinois economy negatively. Nevertheless, the negative effects of an ageing population are smaller in the endogenous model than the forecasts from an exogenous model. This gap may be attributed to the fact that the exogenous model overlooks the roles of human capital stock. An individual has a motive to invest in his/her human capital with a sacrifice of current loss of labor income since individuals expect that his/her current investment in education will pay off more in the future than a current sacrifice.

Two kinds of transfer system as government policy measures were explored under the same scale of budget: money transfer and educational transfer systems. The educational transfer system is superior to money transfer system in the long-run in terms of growth of per-capita income, welfare and stabilizing the factor prices. However, it should be noted that, in the short-run, the drawbacks of the educational transfer system, that is decrease of labor time-share, dominated the positive effects of this transfer system (promotion of human capital stock) since it takes time for human capital stock to be built up enough to cover the impact of the policy's drawbacks. Also, policy-makers should note that there exists a trade-off in implementing transfer policies between economic growth and equality of income and wealth distribution.

The analysis has shown that there is room for government to help weaken and prevent the negative effects of ageing population. Thus, government should pay careful attention to its current fiscal condition so that it could play active role in addressing the challenge of population ageing in the near future. Also the long-term cost/benefits and appropriateness of volumes of directly redistributive policies should be sensibly re-examined ahead of the era of population ageing.

Several points should be mentioned for further study regarding the subjects this paper tried to deal with. First of all, uncertainty factor should be included in the model so that we could draw more realistic implications for government's policy. For example, in last section where there is a transition of inequality measures, it was assumed that human capital stock, asset holding, consumption and time-shares of labor and schooling are identical if the agents are in the same age-cohort in the same year. If individual uncertainty factors such as productivity shock and lay-off risks had been included in the model (then, we can call this model as heterogeneous stochastic general equilibrium model), the individual's reaction to the policy would have been different from those presented. Also, the specification of government policy could be modified to reflect actual policy scheme[16] so that it would be possible to derive more applicable results.

Secondly, another simulation could be implemented to find out the optimal mixture of educational transfer and money transfer for mitigating the negative effects on economic growth[17] and enhancing the equity of the economy as well.

Thirdly, the paper should be developed into one that incorporates the changes of migration pattern and the productivity-heterogeneity of agents. It was assumed that the productivities of agents in the same generation group are identical in this paper. However, in reality, the productivity, that is called human capital stock in this paper, is extensively different across races and immigration status even though they are in the same generations. Also, when government's policy is expected to increase the economic growth, the population structure will become different from that of baseline projection we extensively adopted in this paper since the impact of government policy could cause changes of dynamic migration pattern.

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[16] For example, money transfer could be given proportionally to agent's (revealed) productivity and income and tax could be imposed progressively to the agents according to the income level.

[17] Of course, the objective of economic policy should be different, for example, the objective of transfer system could be decreasing of deep concentration of wealth. However, in this paper, we supposed that the government's primary objective is to boost the economy growth in terms of per-capita output.

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Table 1: Projection of dependency ratio of the U. S and state of Illinois

	2004	2010	2015	2020	2025	2030	2035	2040	2045	2050
U. S.	0.1858	0.1938	0.2196	0.2509	0.2868	0.3156	0.3277	0.3298	0.3295	0.3330
Illinois	0.1793	0.1851	0.2068	0.2338	0.2660	0.2910	..	..	..	..

Source: U. S Census Bureau

Table 2: Key parameter values

Parameter	Description	Value	Source
A	Total factor productivity	1.005	Park and Hewings (2007)
γ	Inverse of intertemporal elasticity of substitution	1.91	Park and Hewings (2007)
J	Life span	65	Park and Hewings (2007)
j^*	Retirement age (=age of working last)	45	Park and Hewings (2007)
θ	Degree of educational investment motive	.012	Sadahiro and Shimasawa (2002)
β	Subjective discount factor	1.011	Park and Hewings (2007)
α	Physical capital income share	.34	Park and Hewings (2007)
B	Efficiency of human capital accumulation	.28	Sadahiro and Shimasawa (2002)
m	Ratio of physical capital stock	.10	Sadahiro and Shimasawa (2002)
π^{hc}	Efficiency of human capital transmission	1.0	Sadahiro and Shimasawa (2002)
ξ	Replacement ratio	.50	Park and Hewings (2007)

Table 3: Growth rate of per-capita output and contributions of its components (%, %p)

	2002-2010	2011-2020	2021-2030	2031-2040	2041-2050	2002-2050
Endogenous case						
Output	1.08	0.44	0.12	0.38	0.47	0.48
(Physical capital stock)	0.24	0.23	0.04	0.10	0.21	0.16
(Labor force)	0.03	-0.27	-0.29	0.03	0.06	-0.09
(Total productivity)	0.81	0.47	0.37	0.26	0.21	0.41
Exogenous case						
Output	0.43	-0.20	-0.55	-0.19	0.02	-0.11
(Physical capital stock)	0.41	0.07	-0.24	-0.21	-0.04	-0.01
(Labor force)	0.03	-0.27	-0.29	0.03	0.06	-0.09

Note: every number is the mean value of each corresponding period.

Table 4: Ratio of welfare between workers and retirees

	2001-2010	2011-2020	2021-2030	2031-2040	2041-2050	2001-2050
Endogenous model	0.89	0.67	0.59	0.61	0.67	0.69
Exogenous model	1.05	0.76	0.63	0.61	0.61	0.73

Note: 1. each number is a mean value during corresponding period.
2. Ratio = average welfare of workers / average welfare of retirees

Table 5: Percentage gap of average labor productivity from the baseline (%)

Scenario	Retirement age	2001	2010	2020	2030	2040	2050
Extending 1 year	66	-0.12	-0.17	-0.40	-0.61	-0.69	-0.63
Extending 3 years	68	-0.44	-0.48	-1.20	-1.82	-2.08	-1.94
Extending 5 years	70	-0.86	-0.77	-2.00	-2.99	-3.46	-3.42

Table 6: Change of average time-share of human capital investment from the baseline model (%)

		2001-2010	2011-2020	2021-2030	2031-2040	2041-2050	2001-2050
Educational transfer							
	10%	6.41	6.84	7.31	7.48	7.48	7.10
	20%	14.35	15.36	16.54	17.00	16.95	16.04
	30%	24.55	26.55	28.88	29.77	29.63	27.87
Money transfer							
	10%	0.72	0.76	0.82	0.88	0.86	0.81
	20%	1.57	1.70	1.87	1.96	1.93	1.80
	30%	2.61	2.85	3.21	3.37	3.37	3.08

Note: In case of money transfer, 10% (20%, 30%) means that total amount of money transfer from government to the household sector is same as that of educational transfer whose reimbursement rate is 10% (20%, 30%).

Table 7: Change of physical capital-labor[18] ratio from the baseline model (%)

	2001-2010	2011-2020	2021-2030	2031-2040	2041-2050	2001-2050
Educational transfer						
10%	-0.13	-0.72	-0.71	-0.62	-0.50	-0.54
20%	-0.24	-1.53	-1.52	-1.33	-1.08	-1.14
30%	-0.32	-2.44	-2.45	-2.18	-1.77	-1.83
Money transfer						
10%	-0.04	-0.13	-0.13	-0.12	-0.11	-0.11
20%	-0.09	-0.29	-0.29	-0.28	-0.26	-0.24
30%	-0.16	-0.48	-0.50	-0.48	-0.46	-0.42

Note: In case of money transfer, 10% (20%, 30%) means that total amount of money transfer from government to the household sector is same as that of educational transfer whose reimbursement rate is 10% (20%, 30%).

Table 8: Change of aggregate welfare from the baseline model (%)

	2001-2010	2011-2020	2021-2030	2031-2040	2041-2050	2001-2050
Educational transfer						
10%	0.83	1.09	1.38	1.63	1.84	1.36
20%	1.73	2.29	2.90	3.46	3.92	2.86
30%	2.70	3.60	4.62	5.53	6.29	4.55
Money transfer						
10%	0.18	0.18	0.20	0.22	0.22	0.20
20%	0.37	0.39	0.44	0.48	0.50	0.44
30%	0.62	0.65	0.74	0.81	0.84	0.73

Note: In case of money transfer, 10% (20%, 30%) means that total amount of money transfer from government to individuals is same as that of educational transfer whose reimbursement rate is 10% (20%, 30%).

[18] Productivity-adjusted labor.

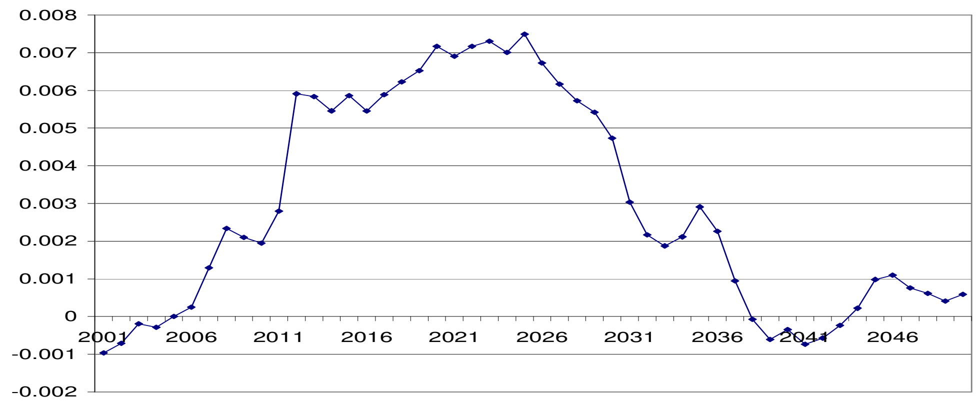


Figure 1: Growth of dependency ratio of the U. S

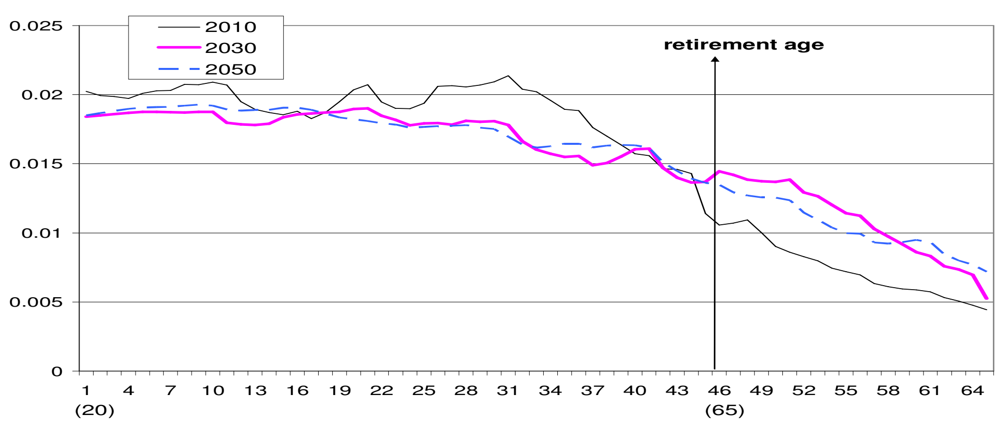


Figure 2**:** Demographic change of Illinois: change of age-cohort structure

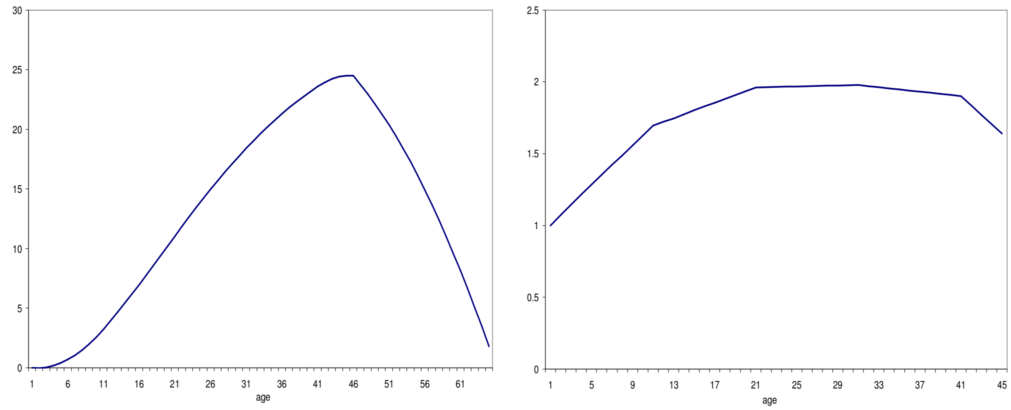


Figure 3: Initial distribution of wealth (left) and human capital stock (right)

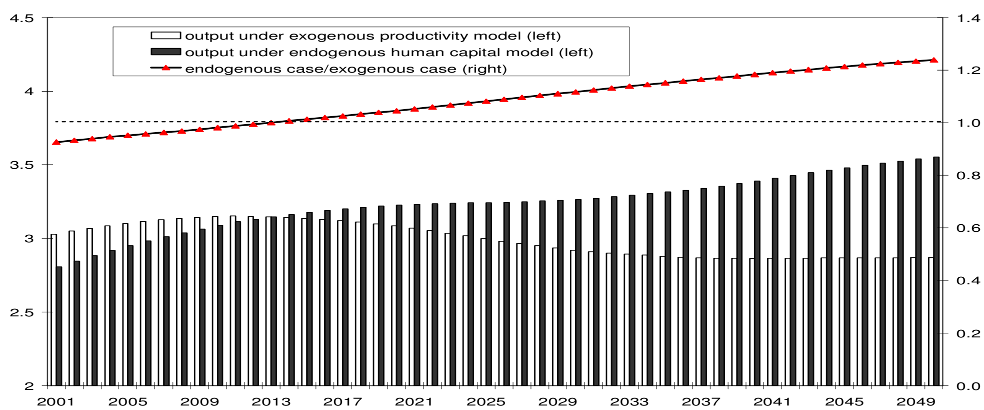
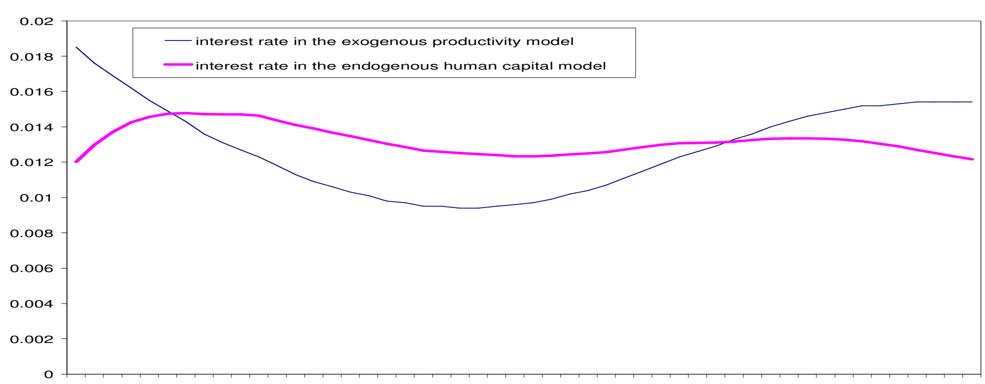


Figure 4: Per-capita output of Illinois in two different model specifications



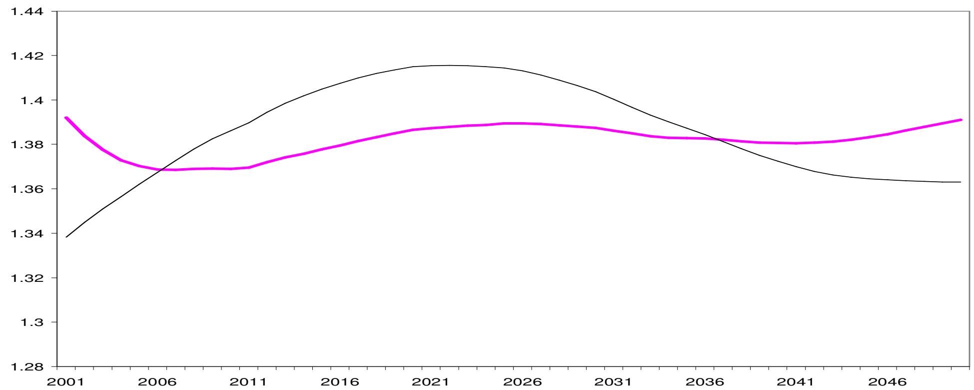


Figure 5: Transition of interest rate (upper) and wage rate

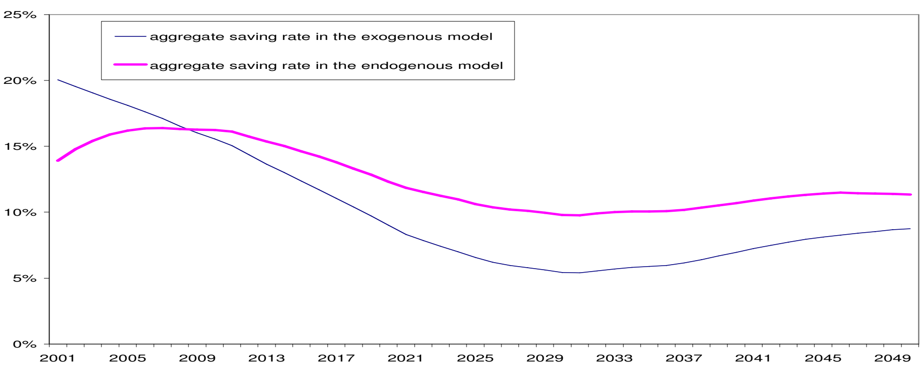


Figure 6: Transition of aggregate saving rate

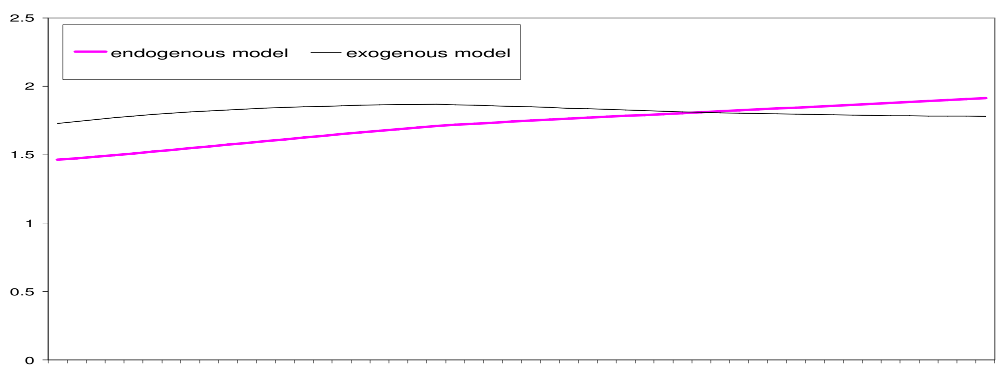


Figure 7: Consumption level corresponding to aggregate social welfare

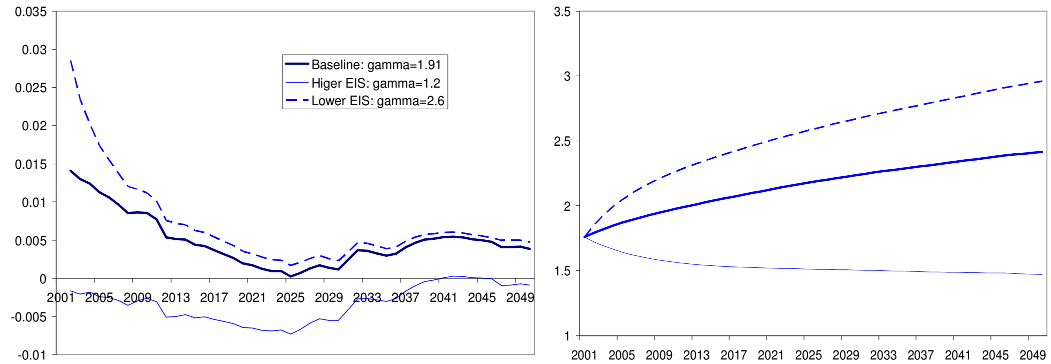


Figure 8: Growth rate of per-capita (left) and transition of average productivity (human capital stock) of workers under different EIS ($1/\gamma$) value assignment

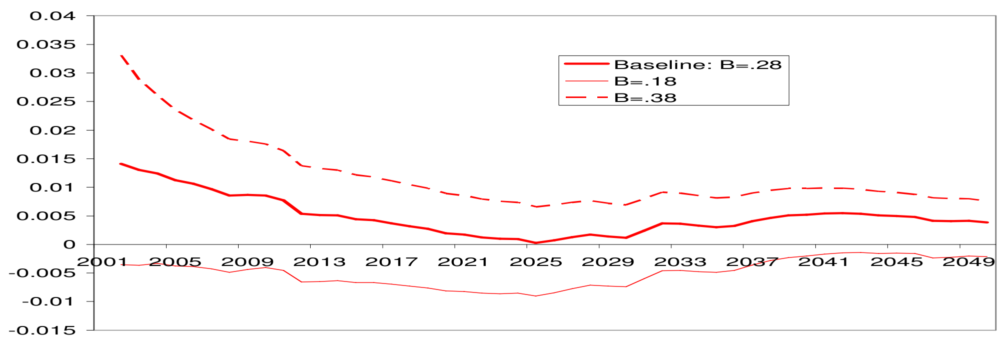


Figure 9: Growth rate of per-capita output under different parameter values B

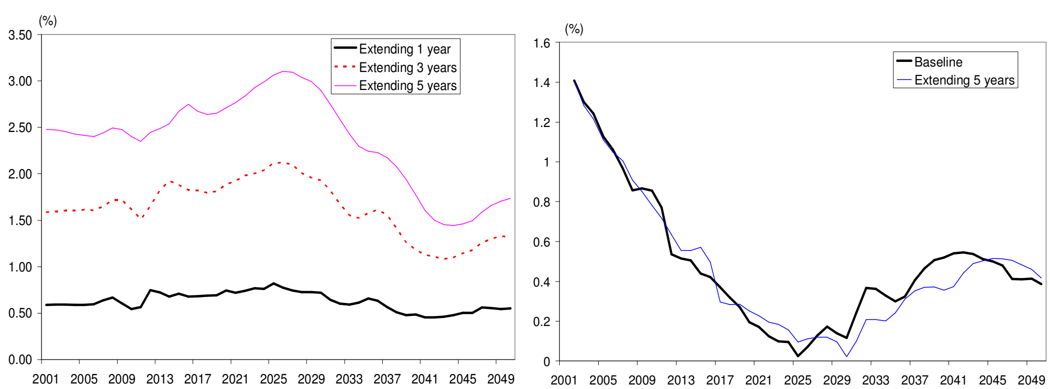


Figure 10: Percentage gap from the baseline (left) and growth rate (right) of the per-capita output when extending retirement age

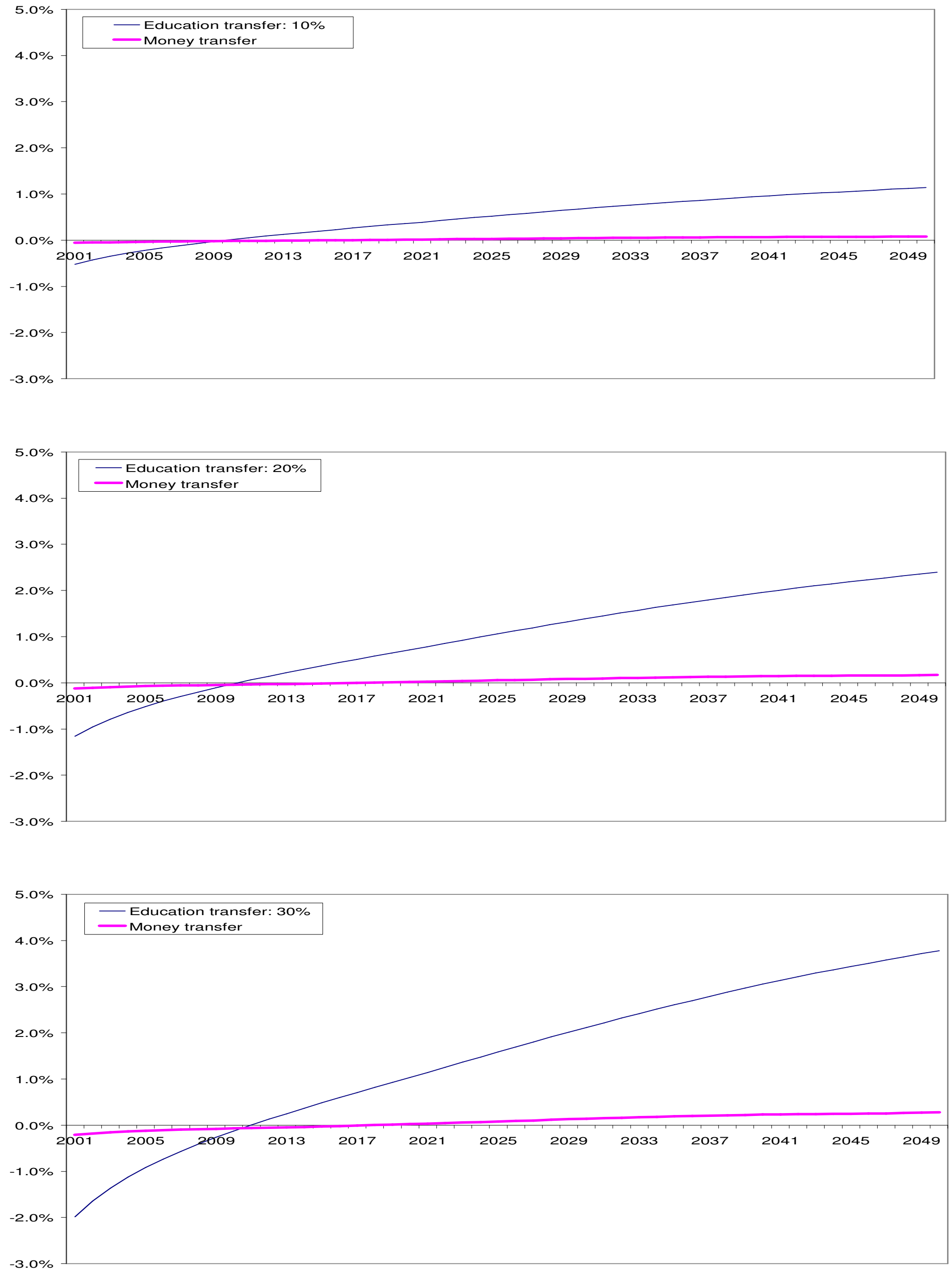


Figure 11: Change of per-capita output from the baseline model

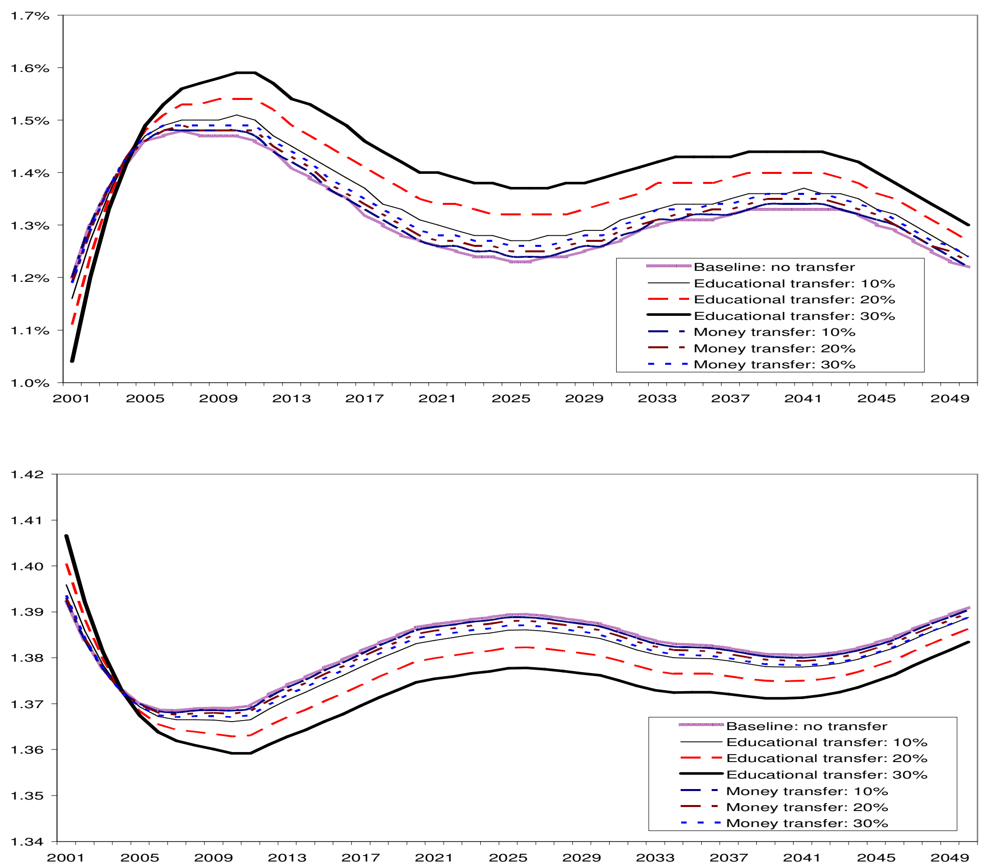


Figure 12: Transition of interest rate (upper) and wage rate (lower)

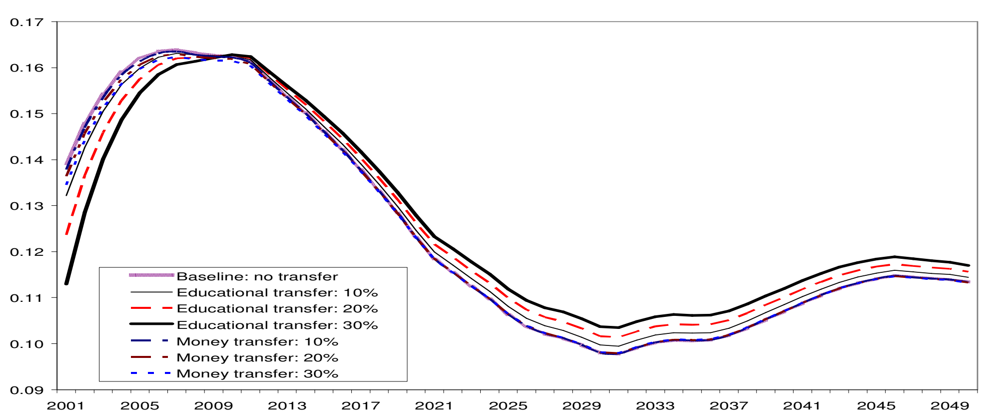


Figure 13**:** Transition of aggregate saving rate

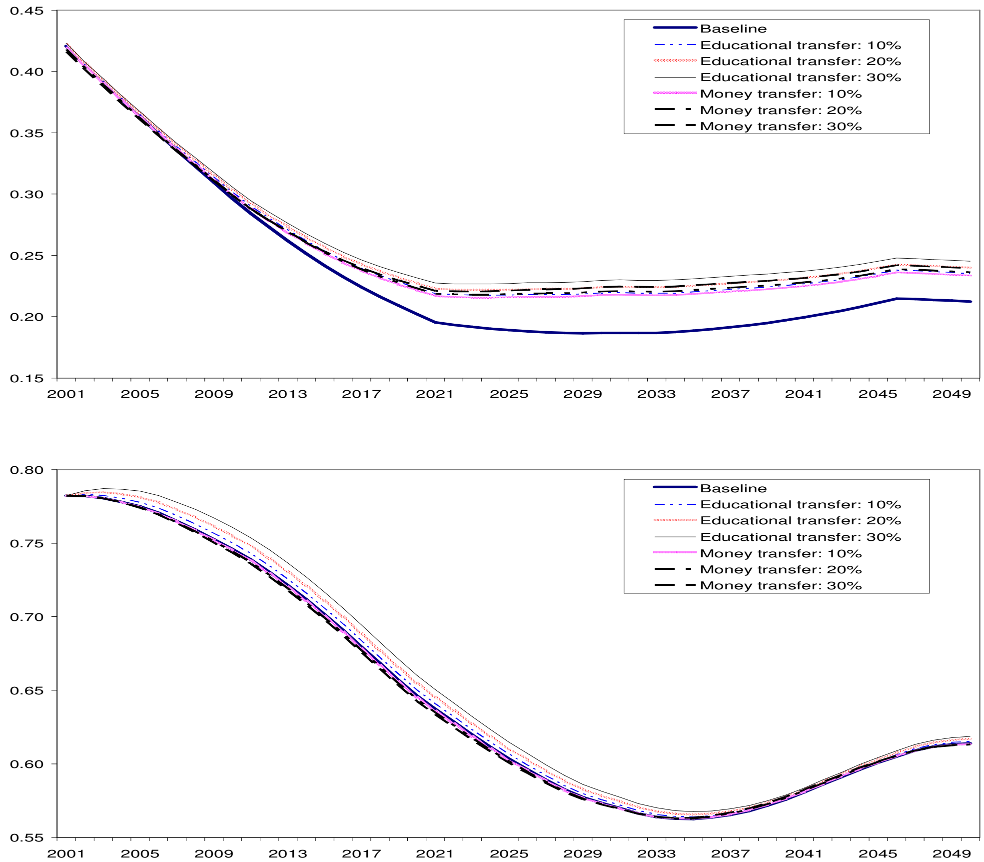


Figure 14: Gini coefficient of income (upper) and wealth